ANALYSIS OF A SEMI-IMPLICIT PROJECTION METHOD FOR THE INCOMPRESSIBLE NAVIER-STOKES EQUATIONS

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Abstract. This paper considers a second-order semi-implicit projection method for the incompressible viscous flows. In the projection method, the Crank-Nicolson scheme is used for both the advection and diffusion terms. Based on a block LU decomposition, velocity and pressure are decoupled(pressure-velocity decoupling) preserving the temporal second-order accuracy. Then the momentum equations can be treated by two ways. First way is iterative semi-implicit method, which means the momentum equations for the intermediate velocity are able to be calculated by using iterative schemes. For another treatment, we call it decoupling semi-implicit method, the intermediate velocity components are additionally decoupled(velocity-velocity decoupling) by using an approximate factorization without any modification of boundary conditions, which enables the decoupled momentum equations can be solved without any iterations. These methods have been widely used for various turbulence problems efficiently and feasibly. However, the stability and order of accuracy were only proved upon numerical simulation results.

The present study is focused on a mathematical analysis on the stability and accuracy of the iterative semi-implicit projection method. Three kinds of formulations of advection term will be involved in the following discussion. First, we show that the projection method is globally stable for by estimating the kinetic energy. And considering a perturbed semi-discretized equations of the projection method, we perform von Neumann stability analysis. The stability criteria is obtained by calculating the eigenvalue analysis of system matrix. Unconditional stability property can be observed for the iterative semi-implicit projection method. We also show the iterative semi-implicit projection method leading to a solution with $O(\triangle t^2)$ accuracy in both the velocity and pressure for solving the unsteady incompressible Navier-Stokes equations. Finally, we perform simulations for the von Neumann analysis and well-established benchmark problems, validating the present stability and accuracy analysis.

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