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Two-Fluid Models

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Outline

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- Interpenetrating Continua
- Two-Fluid Models
- Governing Equations
- Computational Modeling
- Applications
- Conclusions

Single Phase Flows

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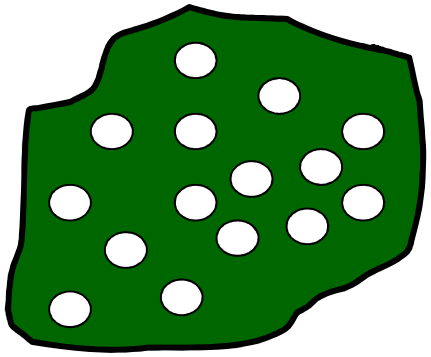
Continuum Assumption



3×10^{19} molecules in $1\ \text{cm}^3$

Fluid is a Continuum

Density:

$$\rho = \lim_{L \to 0} \frac{\sum m_i}{V}$$

Velocity (mass-averaged)

$$\mathbf{v} = \lim_{L \to 0} \frac{\sum m_i \mathbf{v}_i}{\sum m_i}$$

Multiphase Flows

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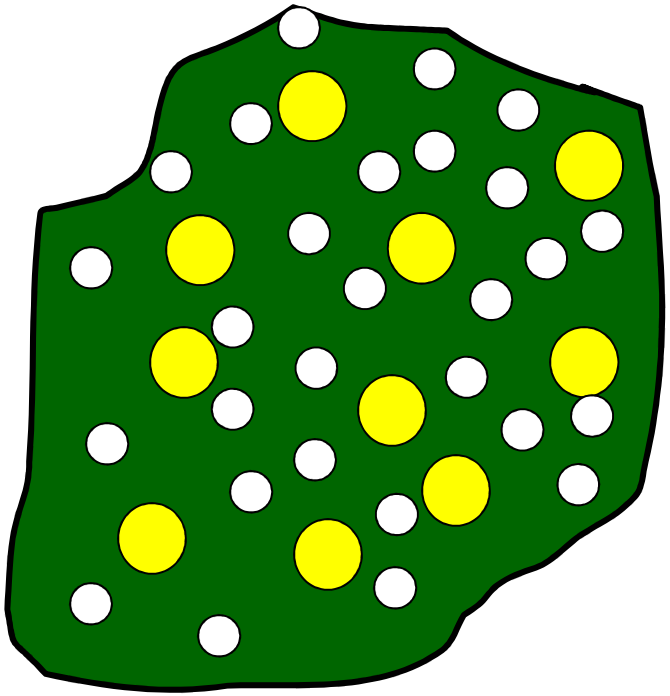
Interpenetrating Continua



Density of constituent α

$$\rho^{\alpha} = \lim_{L \to 0} \frac{\sum m_i^{\alpha}}{V}$$

Velocity of αth phase

$$\mathbf{v}^{\alpha} = \lim_{L \to 0} \frac{\sum m_i^{\alpha} \mathbf{v}_i^{\alpha}}{\sum m_i^{\alpha}}$$

Mass Conservation

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Gas Phase

$$\frac{\partial}{\partial t}(\alpha_g \rho_g) + \nabla \cdot (\alpha_g \rho_g \vec{V}_g) = 0$$

Solid Phase

$$\frac{\partial}{\partial t}(\alpha_s \rho_s) + \nabla \cdot (\alpha_s \rho_s \vec{V}_s) = 0$$

$$\rho_g = \frac{P_g M_w}{R T_g}$$

Volume Fraction

$$\alpha_s + \alpha_g = 1$$

Balance of Momentum

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Gas Phase

$$\frac{\partial}{\partial t}(\alpha_g \rho_g \vec{V}_g) + \nabla \cdot (\alpha_g \rho_g \vec{V}_g \vec{V}_g) = \nabla \cdot \bar{\bar{\tau}}_g + \alpha_g \rho_g \vec{g} + \alpha_s \nabla P_g + \beta\left(\vec{V}_s - \vec{V}_g\right)$$

Solid Phase

$$\frac{\partial}{\partial t}(\alpha_s \rho_s \vec{V}_s) + \nabla .(\alpha_s \rho_s \vec{V}_s \vec{V}_s) = \nabla \cdot \bar{\bar{\tau}}_s + \alpha_s \rho_s \vec{g} - \alpha_s \nabla P_g + \beta\left(\vec{V}_g - \vec{V}_s\right)$$

Stress Tensor

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Gas phase turbulence stress tensor

$$\overline{\overline{\tau}}_g = -P_g \overline{\overline{I}} + 2\alpha_g \mu_g \overline{\overline{D}}_g + \alpha_g \lambda_g tr(\overline{\overline{D}}_g)\overline{\overline{I}}$$

$$\overline{\overline{D}}_g = \frac{1}{2}[\nabla \vec{V}_g + (\nabla \vec{V}_g)^T]$$

Solid phase turbulence stress tensor

$$\overline{\overline{\tau}}_s = -\nabla P_s + 2\mu_s \overline{\overline{D}}_s + \eta\lambda_s tr(\overline{\overline{D}}_s)\overline{\overline{I}}$$

$$\overline{\overline{D}}_s = \frac{1}{2}[\nabla \vec{V}_s + (\nabla \vec{V}_s)^T]$$

$$P_s = \alpha_s \rho_s \Theta_s [1 + 2\alpha_s g_0 (1+e)]$$

Effective Viscosities

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$$\mu_g = \mu_g^v + \mu_g^t$$

$$\mu_g^t = \rho_g C_\mu \frac{k_1}{\varepsilon_1}$$

$$\mu_s = \alpha_s \rho_s \left(\upsilon_s^{kin} + \upsilon_s^{col} \right)$$

$$\lambda_s = \frac{5}{3} \alpha_s \rho_s \upsilon_2^{col}$$

$$\upsilon_2^{kin} = \left[\frac{2}{3} k_{12} \eta_t + \Theta_s \left(1 + \zeta_{c2} \alpha_s g_0 \right) \right] \tau_2$$

$$\upsilon_2^{col} = \frac{8}{5} \alpha_s g_0 \frac{(1+e)}{2} \left(\upsilon_2^{kin} + d_p \sqrt{\frac{\Theta_s}{\pi}} \right)$$

Drag Coefficient

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$$\beta_{Ergun} = 150\frac{\alpha_s \mu_g}{\alpha_g d_{ps}^2} + 1.75\frac{\alpha_s \rho_g}{d_{ps}}\left|\vec{V}_s - \vec{V}_g\right| \qquad \alpha_g < 0.8$$

$$\beta_{Wen-Yu} = \frac{3}{4} C_D \frac{\alpha_s \rho_g}{d_{ps}}\left|\vec{V}_s - \vec{V}_g\right| \alpha_g^{-2.65} \qquad \alpha_g \geq 0.8$$

$$\beta_{Cao-Ahmadi} = \frac{18\mu_g \alpha_s}{d_s^2} \frac{[1 + 0.1(\mathrm{Re}_s)^{0.75}]}{\left(1 - (\frac{\alpha_s}{\alpha_{s,\max}})\right)^{2.5\alpha_{s,\max}}}$$

$$C_D = \begin{cases} \dfrac{24}{\alpha_g \mathrm{Re}_p}[1 + 0.15(\alpha_g \mathrm{Re}_p)^{0.687}] & \mathrm{Re}_p < 1000 \\ 0.44 & \mathrm{Re}_p \geq 1000 \end{cases}$$

$$\mathrm{Re}_p = \frac{d_{ps}\left|\vec{V}_s - \vec{V}_g\right|\rho_g}{\mu_g}$$

Gas Phase Turbulence

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The k-ε Turbulence Model

$$\frac{\partial}{\partial t}\left(\alpha_g \rho_g k_1\right)+\nabla\cdot\left(\alpha_g \rho_g k_1 \vec{V}_g\right)=\nabla\cdot\left(\alpha_g \frac{\mu_{t1}}{\sigma_k}\nabla k_1\right)+\alpha_g \overline{\overline{\tau}}_g : \nabla \vec{V}_g + \Pi_{k1} - \alpha_g \rho_g \varepsilon_1$$

$$\frac{\partial}{\partial t}\left(\alpha_g \rho_g \varepsilon_1\right)+\nabla\cdot\left(\alpha_g \rho_g \varepsilon_1 \vec{V}_g\right)=\nabla\cdot\left(\alpha_g \frac{\mu_{t1}}{\sigma_\varepsilon}\nabla \varepsilon_1\right) + \alpha_g \frac{\varepsilon_1}{k_1}\left(C_{1\varepsilon}\overline{\overline{\tau}}_g : \nabla \vec{V}_g - C_{2\varepsilon}\rho_g \varepsilon_g\right)+\Pi_{\varepsilon 1}$$

Solid Phase Turbulence

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Simonin Model

Gas phase turbulence kinetic Energy

$$k_1 = \frac{1}{2}\vec{V}'_g \cdot \vec{V}'_g$$

$$k_{12} = \vec{V}'_g \cdot \vec{V}'_s$$

Solid phase turbulence kinetic Energy

$$k_2 = \frac{1}{2}\vec{V}'_s \cdot \vec{V}'_s$$

Granular temperature

$$\Theta_s = \frac{2}{3}k_2$$

Solid Phase Turbulence

Clarkson University

Simonin Model

$$\frac{\partial}{\partial t}(\alpha_s \rho_s k_2) + \nabla \cdot \left(\alpha_s \rho_s k_2 \vec{V}_s\right) = \nabla \cdot (\alpha_s K_2 \nabla k_2) + \alpha_s \overline{\overline{\tau}}_s : \nabla \vec{V}_s + \Pi_{k2} - \alpha_s \rho_s \varepsilon_2$$

$$\frac{\partial}{\partial t}(\alpha_s \rho_s k_{12}) + \nabla \cdot \left(\alpha_s \rho_s k_{12} \vec{V}_g\right) = \nabla \cdot \left(\alpha_s \rho_s \frac{\upsilon_{12}^t}{\sigma_k} \nabla k_{12}\right) + \alpha_s \rho_s \left(\tau_{12} \overline{\overline{\tau}}_s : \nabla \vec{V}_s - \varepsilon_{12}\right) + \Pi_{k12}$$

Solid Phase Turbulence

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Simonin Model Parameters

$$\mathrm{K}_2 = \alpha_s \rho_s \left(\mathrm{K}_2^{kin} + \mathrm{K}_2^{col} \right)$$

$$\mathrm{K}_2^{kin} = \left(9/10 k_{12} \eta_t + \frac{3}{2} \Theta_s \left(1 + \varpi_c \alpha_s g_0 \right) \right) \left(\frac{9/5}{\tau_{12}^x} + \frac{\zeta_c}{\tau_2^c} \right)^{-1}$$

$$\mathrm{K}_2^{col} = 18/5 \alpha_s g_0 \frac{(1+e)}{2} \left(\mathrm{K}_2^{kin} + 5/9 d_p \sqrt{\frac{\Theta_s}{\pi}} \right)$$

Solid Phase Turbulence

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Simonin Model Parameters

$$\varepsilon_2 = 12\left(1-e^2\right)\alpha_s^2 \rho_s g_0 \frac{\Theta_s^{3/2}}{d_p}$$

$$\Pi_{k1} = \beta\left(k_{12} - 2k_1\right)$$

$$\Pi_{\varepsilon 1} = C_{3\varepsilon}\left(\varepsilon_1 / k_1\right)\Pi_{k1}$$

$$\Pi_{k2} = \beta\left(k_{12} - 3\Theta_s\right)$$

$$k_{12} = \frac{\eta_t}{1+\left(1+X_{21}\right)\eta_t}\left(2k_1 + 3X_{21}\Theta_s\right)$$

Solid Phase Turbulence

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Ahmadi-Ma Model

Solid phase turbulence kinetic Energy

$$\frac{\partial}{\partial t}\left(\alpha_s \rho_s k_2\right)+\nabla\cdot\left(\alpha_s \rho_s k_2 \vec{V}_s\right)=\nabla\cdot\left(\alpha_s K_2 \nabla k_2\right)$$
$$+\alpha_s \bar{\bar{\tau}}_s : \nabla \vec{V}_s + \Pi_{k2} - \alpha_s \rho_s \varepsilon_2$$

$$\varepsilon_2 = 12\left(1-e^2\right)\alpha_s^2 \rho_s g_0 \frac{\Theta_s^{3/2}}{d_p}$$

Solid Phase Turbulence

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Ahmadi-Ma Model Parameters

$$K_2 = 0.1306\rho_s d_p \left(1+e^2\right)\left(1/g_0 + 4.8\alpha_s + 12.1184 g_0 \alpha_s^2\right)\sqrt{\Theta_s}$$

$$\Pi_{k1} = \beta\left(\Theta_s - 2k_1\right)$$

$$\Pi_{k2} = \beta\left(\frac{2k_1}{1+\tau_{12}^x/\tau_1} - 3\Theta_s\right)$$

Gas Phase Inlet Condition

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$$V_g^{inlet}(r) = \frac{60}{49}\frac{V_g}{1-\alpha_s^{inlet}}\left(1-\frac{2r}{D}\right)^{1/7}$$

$$k_1^{inlet}(r) = 0.004\left(V_g^{inlet}(r)\right)^2$$

$$\varepsilon_1(r) = 2\frac{\left(k_1^{inlet}(r)\right)^{3/4}}{0.4187D}$$

Solid Phase Inlet Condition

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$$V_s^{inlet} = \frac{4M_s}{\alpha_s^{inlet}\rho_s \pi D^2}$$

$$\alpha_s^{inlet} = \frac{M_s}{\rho_s}\left(\frac{M_g}{\rho_g} + \frac{M_s}{\rho_s}\right)^{-1}$$

$$\Theta_s^{inlet} = 0.004\left(V_s^{inlet}\right)^2$$

Solid Phase Wall Boundary Condition

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Slip boundary condition

$$\left.\frac{\mu_s^t}{\rho_s}\frac{\partial V_s}{\partial z}\right|_w + \frac{\phi\pi V_s g_0 \sqrt{2k_2/3}}{2\sqrt{3}\alpha_{s\,\max}} = 0$$

$$K_2\left.\frac{\partial k_2}{\partial z}\right|_w - \frac{\phi\pi V_s g_0 \sqrt{2k_2/3}}{2\sqrt{3}\alpha_{s\,\max}} + \frac{\sqrt{3}\pi g_0 \left(1-e_w^2\right)\left(2k_2/3\right)^{3/2}}{4\alpha_{s\,\max}} = 0$$

Pergamon

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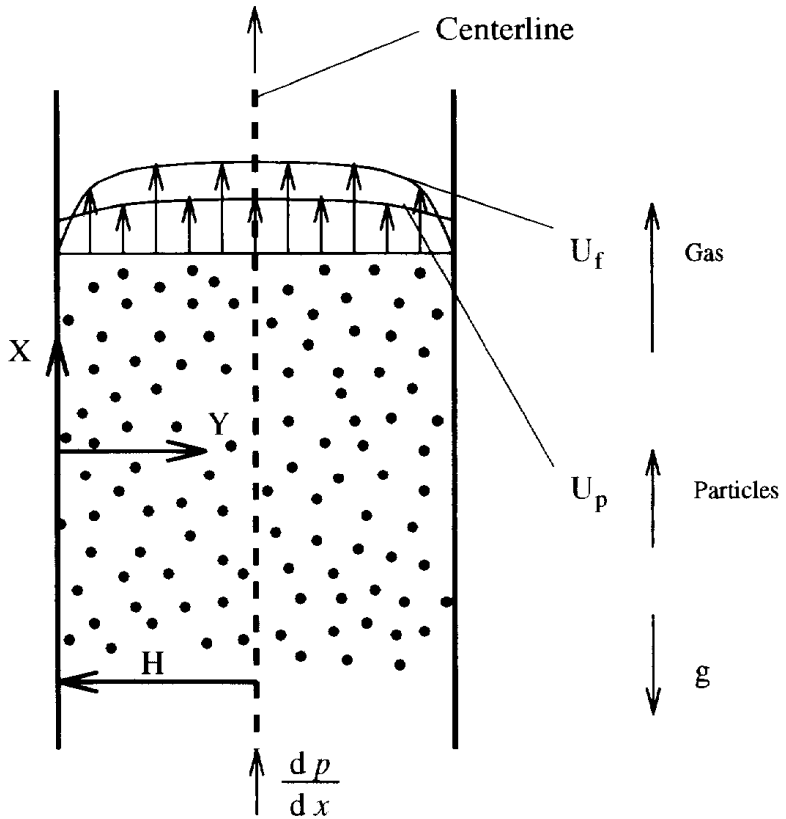
GAS–PARTICLE TWO-PHASE TURBULENT FLOW IN A VERTICAL DUCT

J. CAO and G. AHMADI



Gas Velocity, Turbulence

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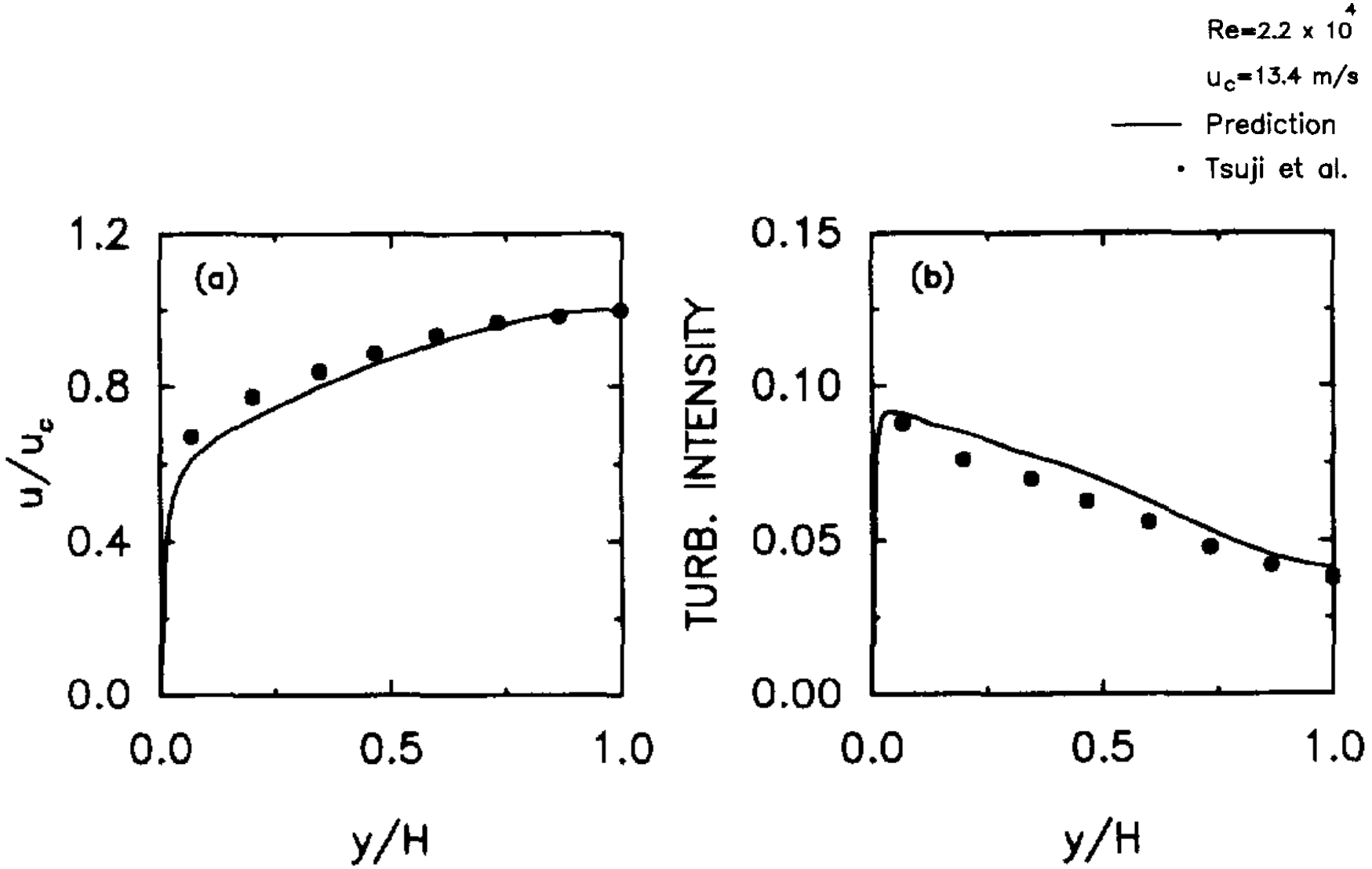
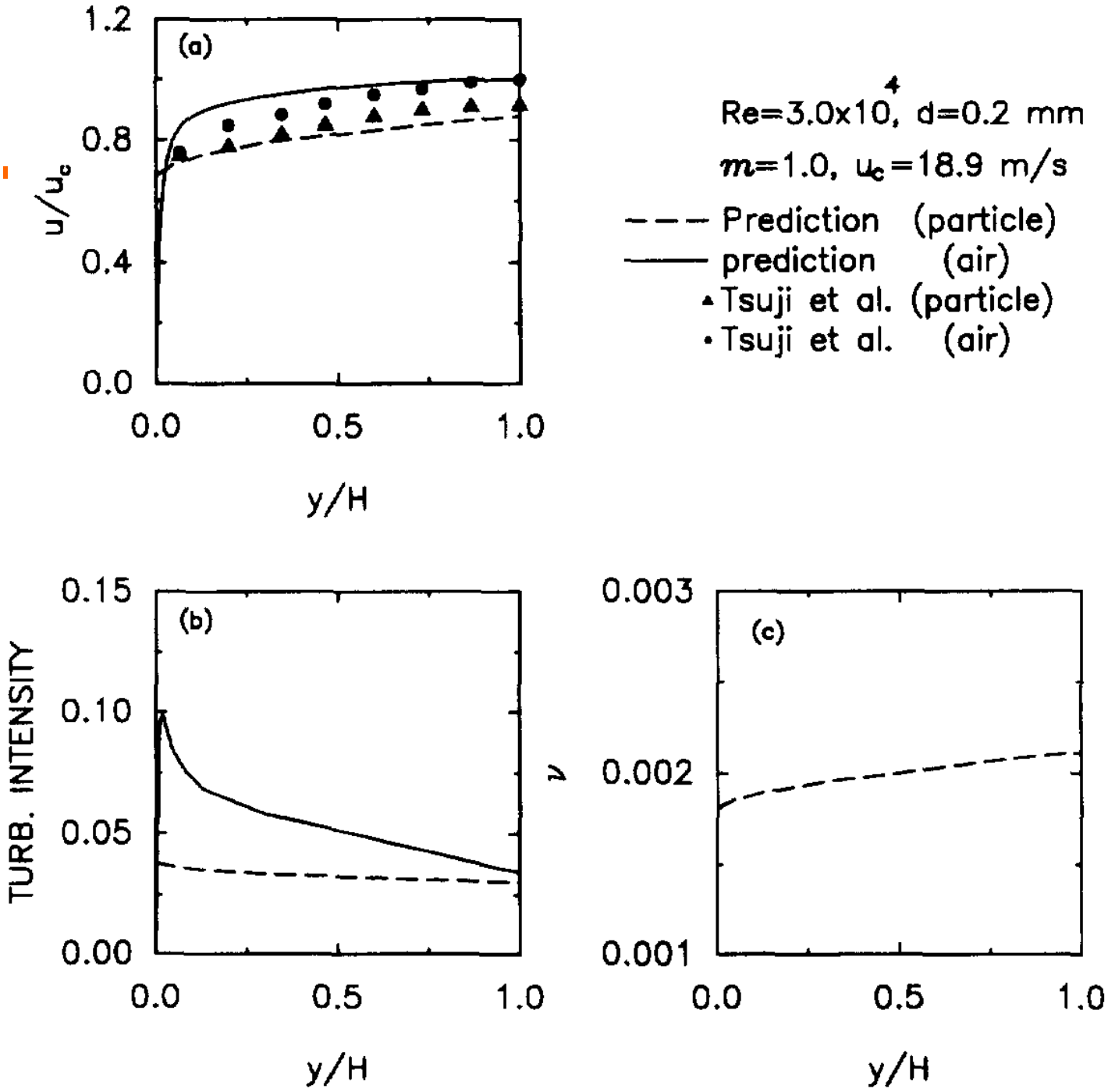
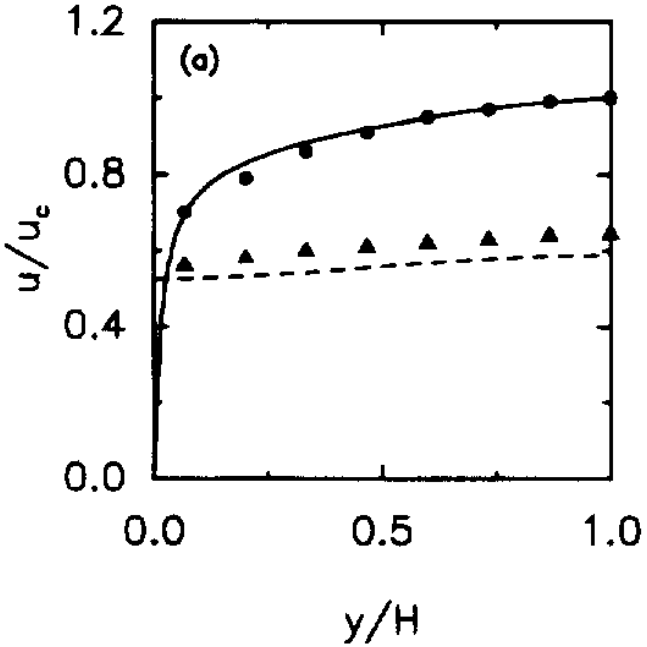



(a)
1.2
0.8
0.4
0.0
u/u_c
0.0
0.5
1.0
y/H
Re=3.0x10^4, d=0.2 mm
m=1.0, u_c=18.9 m/s
Prediction (particle)
prediction (air)
Tsuji et al. (particle)
Tsuji et al. (air)
(b)
0.15
0.10
0.05
0.00
TURB. INTENSITY
0.0
0.5
1.0
y/H
(c)
0.003
0.002
0.001
ν
0.0
0.5
1.0
y/H



Re=1.6x10^4, d=0.5 mm

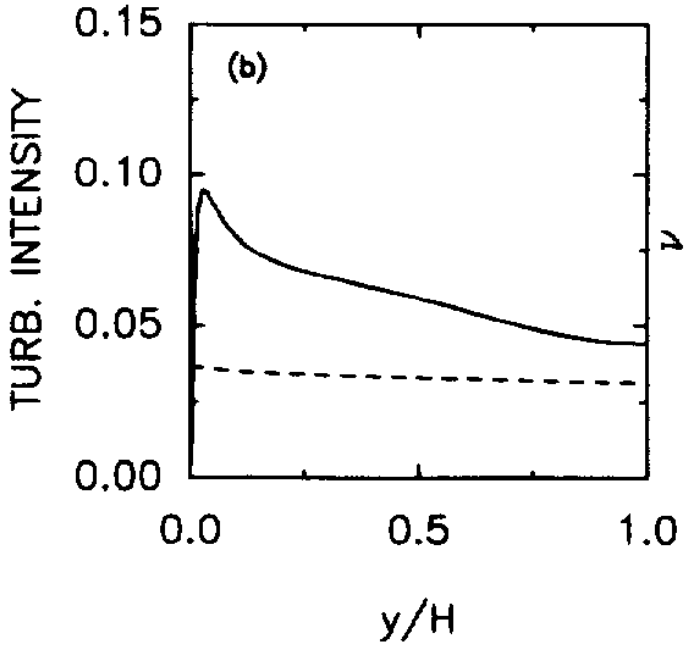
m=1.1, u_c=9.65 m/s

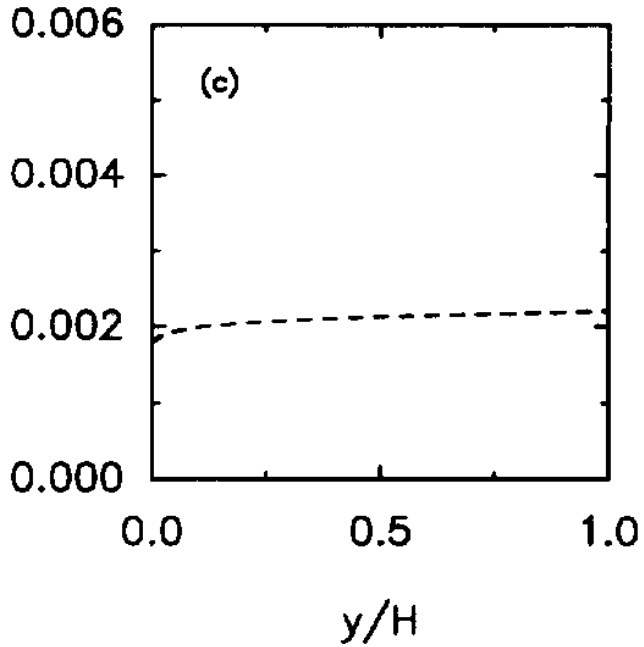
Prediction (particle)

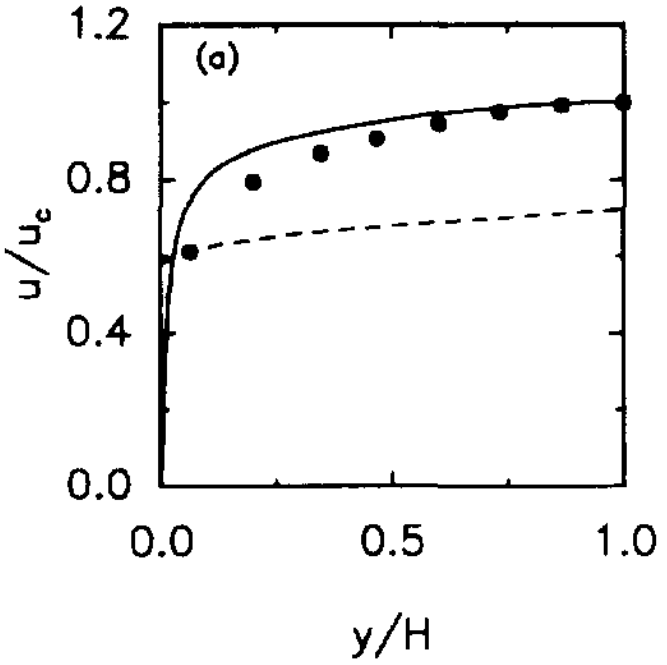
Prediction (air)

Tsuji et al. (particle)

Tsuji et al. (air)

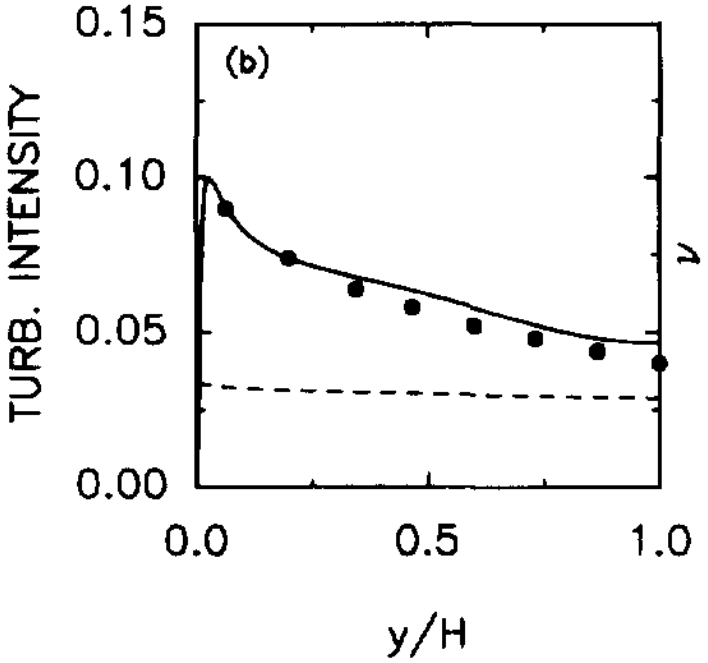


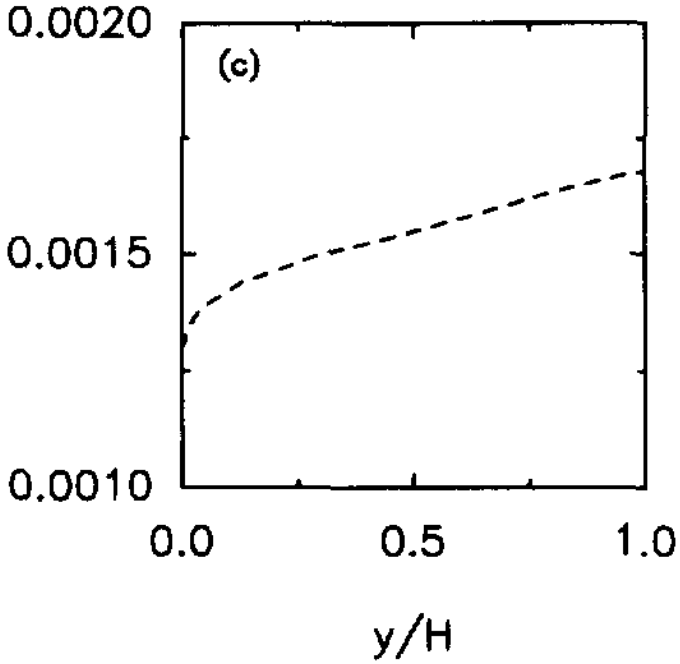


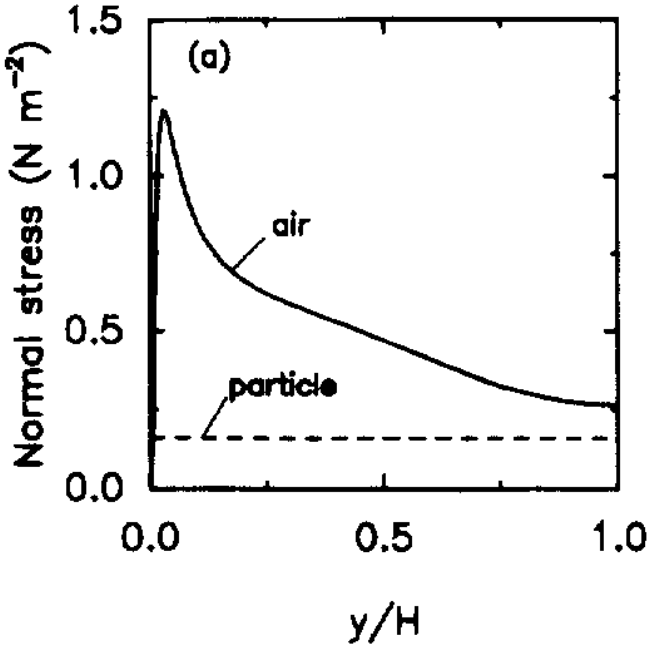


Re=2.2x10^4, d=0.5 mm
m=0.7, u_c=12.2 m/s

----- Prediction (particle)
——— Prediction (air)
• Tsuji et al. (air)



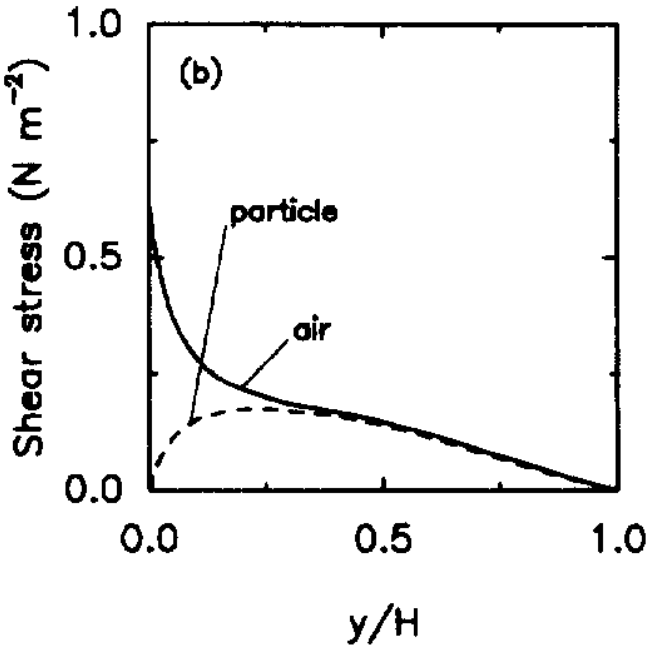


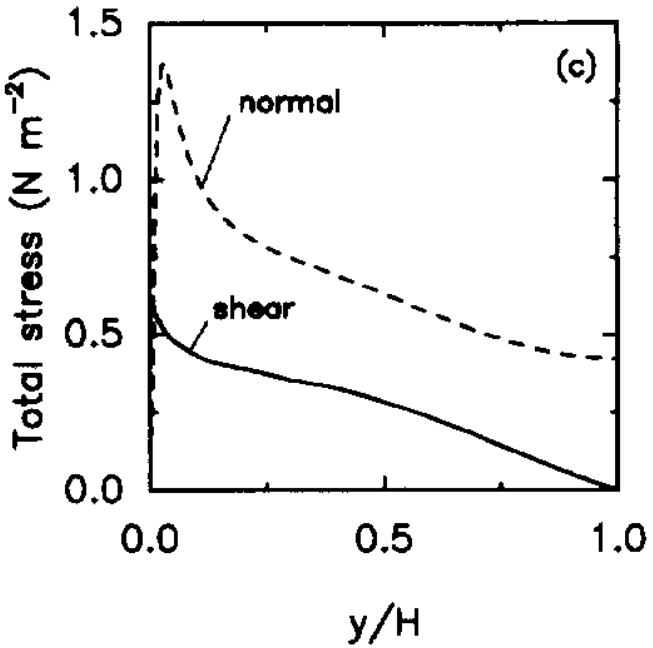


m=0.7

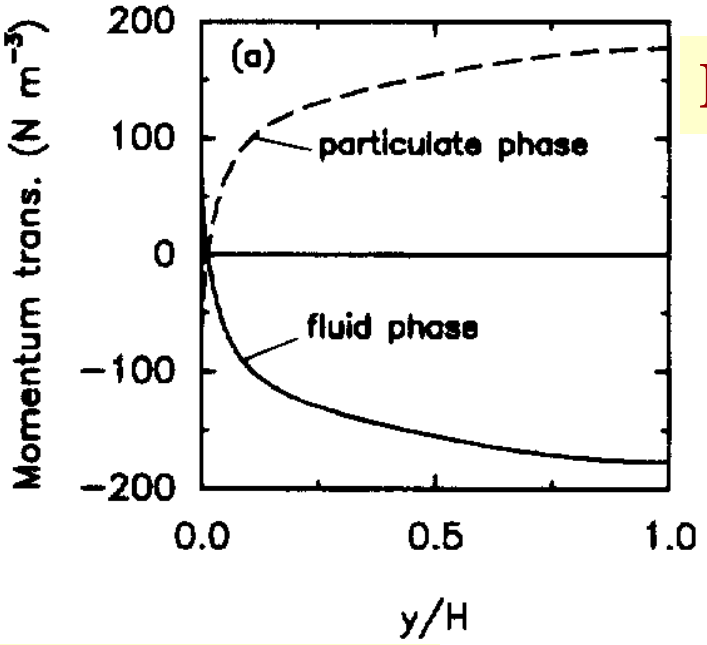
u_c=12.2 m/s

Re=2.2x10^4, d=0.5 mm





Momentum Supply

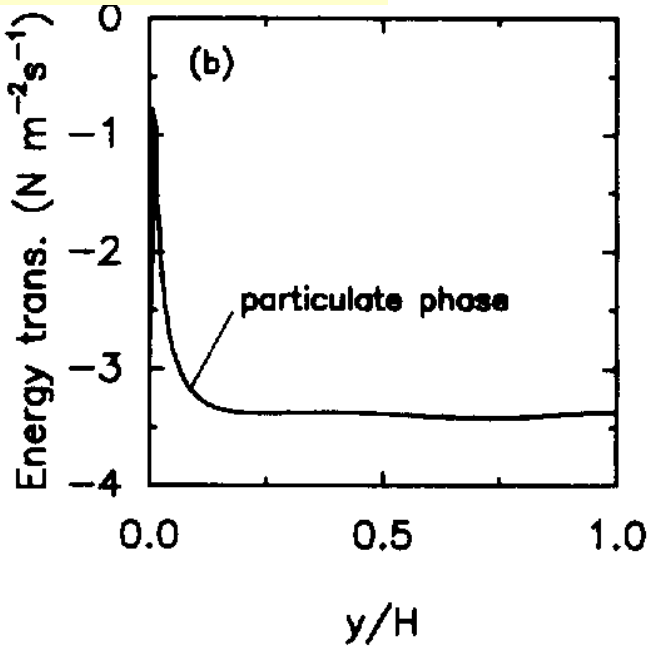


m=0.7

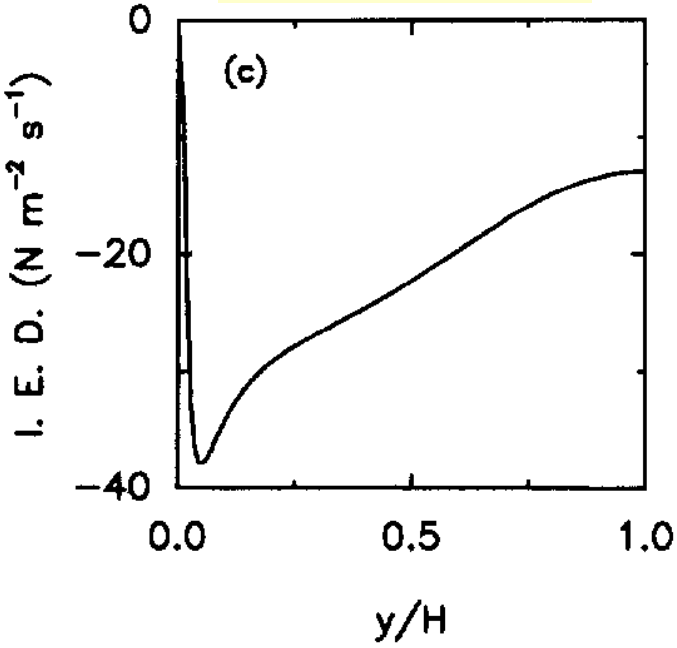
u_c=12.2 m/s

Re=2.2x10^4, d=0.5 mm

Energy Supply



Interaction Dissipation



Energy Production/ Dissipation

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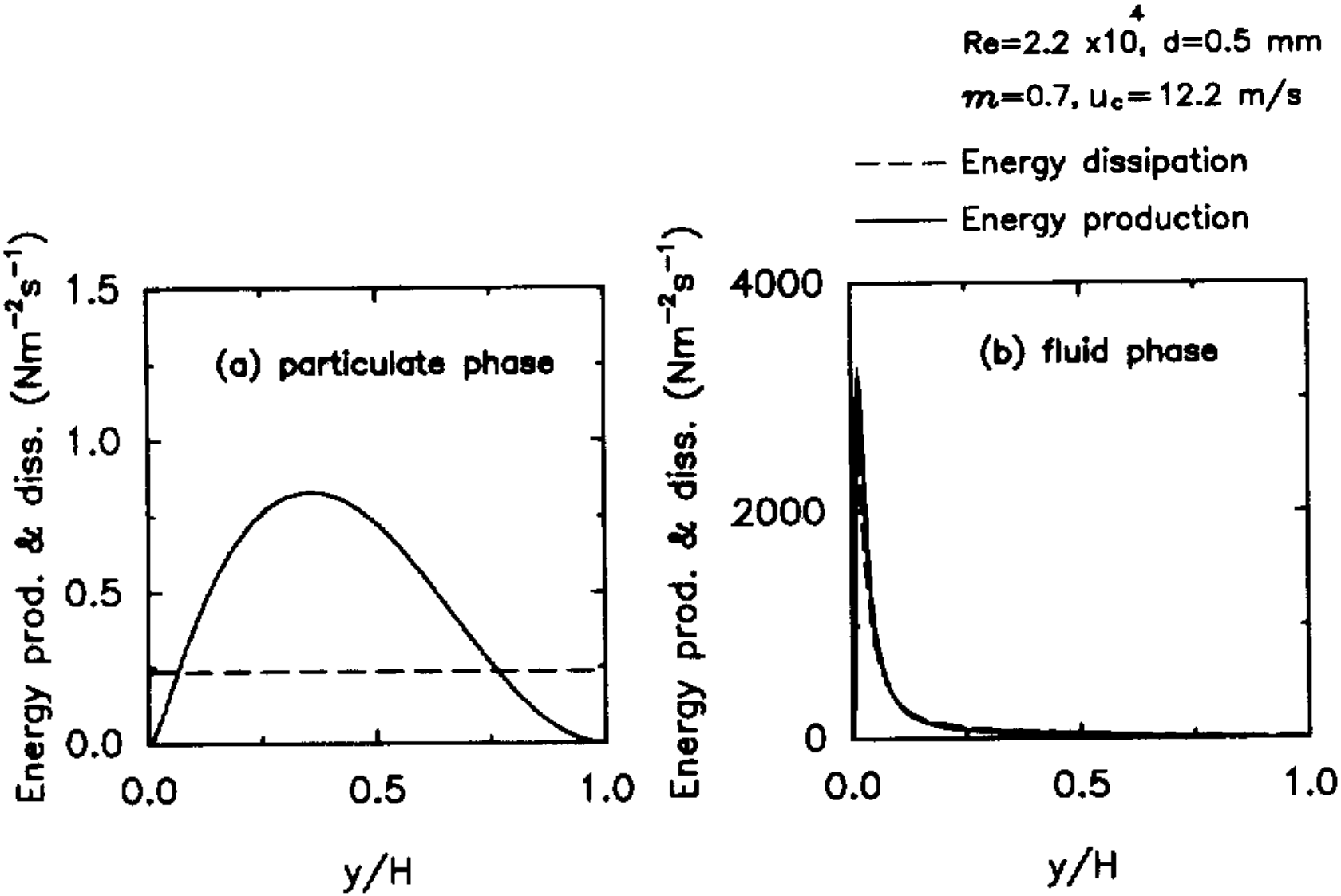
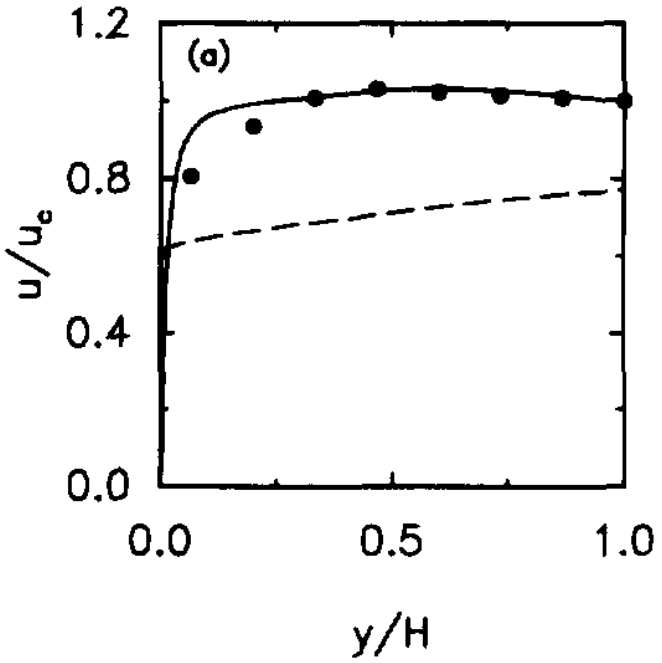




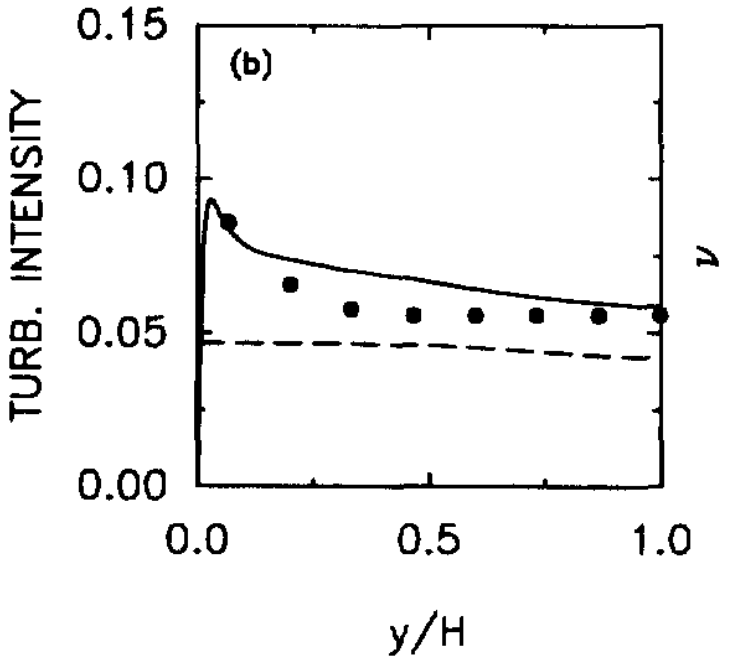
Re=2.2x10^4, d=0.5 mm

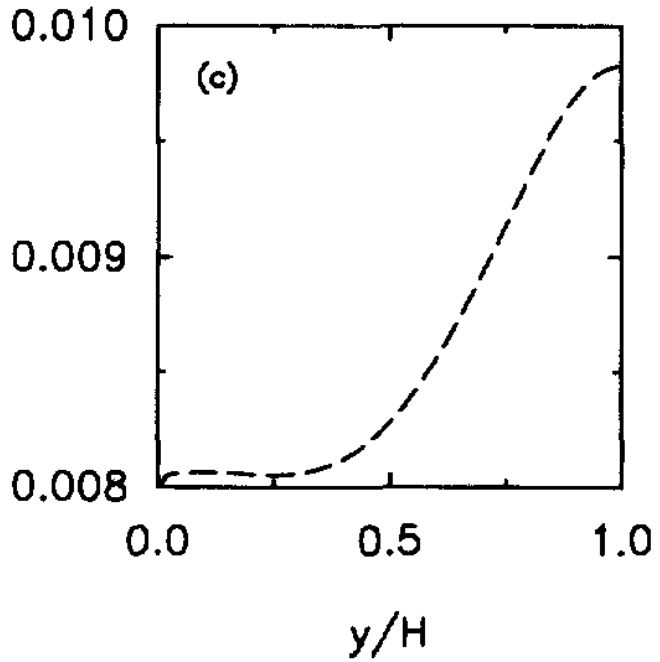
m=3.4, u_c=10.7 m/s

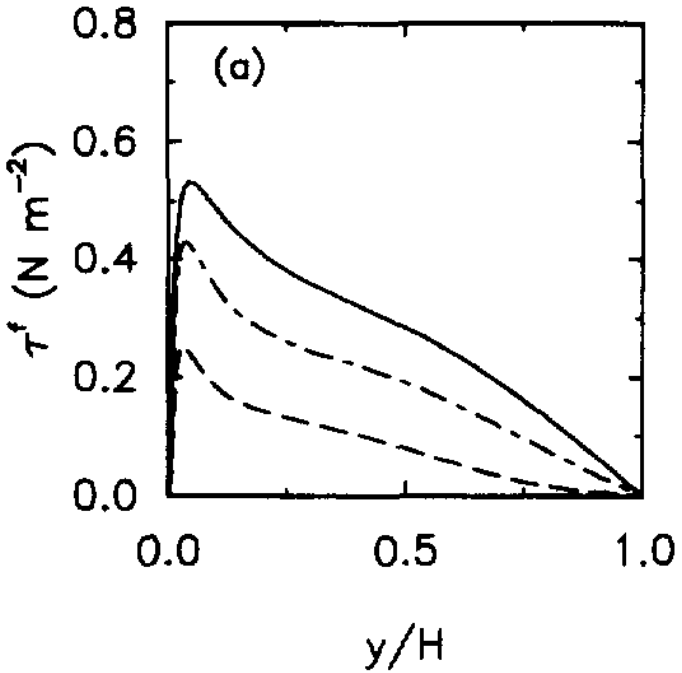
--- Prediction (particle)

—— Prediction (air)

• Tsuji et al. (air)







Re=2.2x10^4

d=0.5 mm

m = 0.0

m = 0.7

m = 3.4

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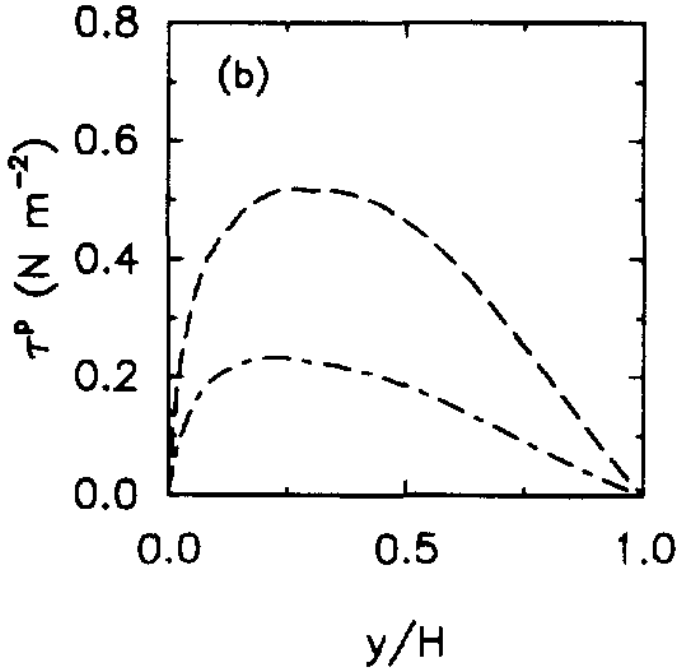
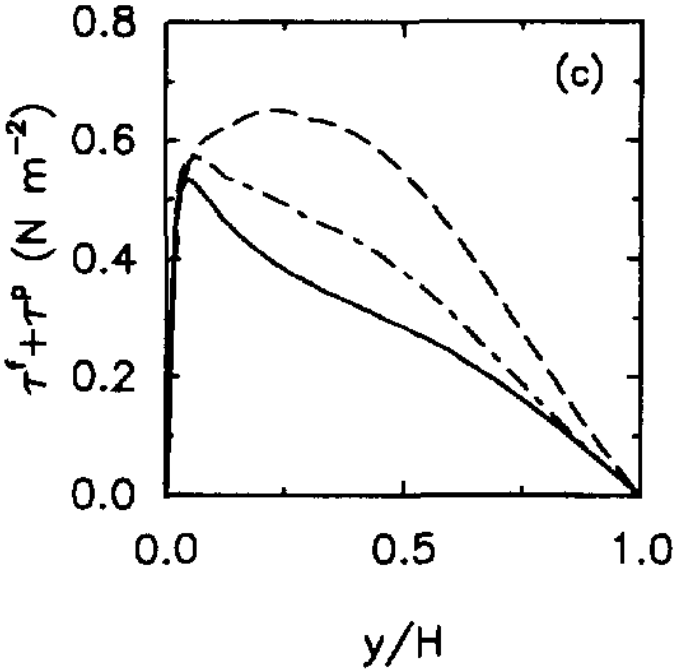
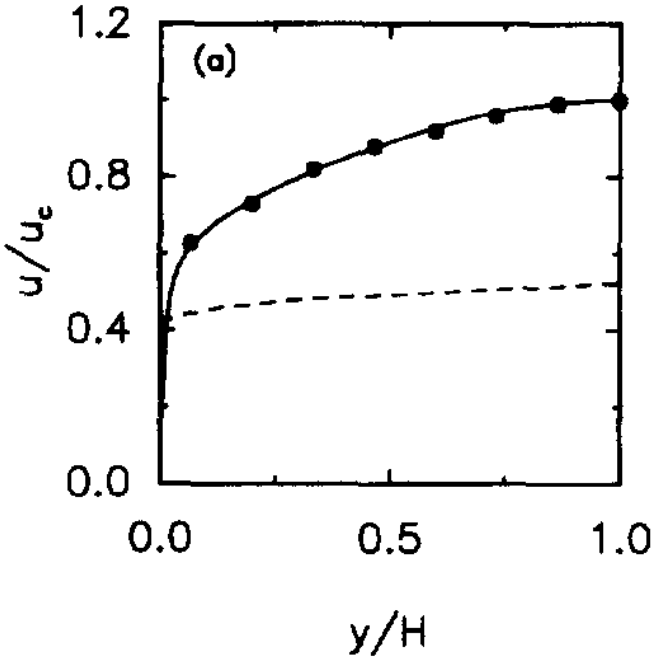






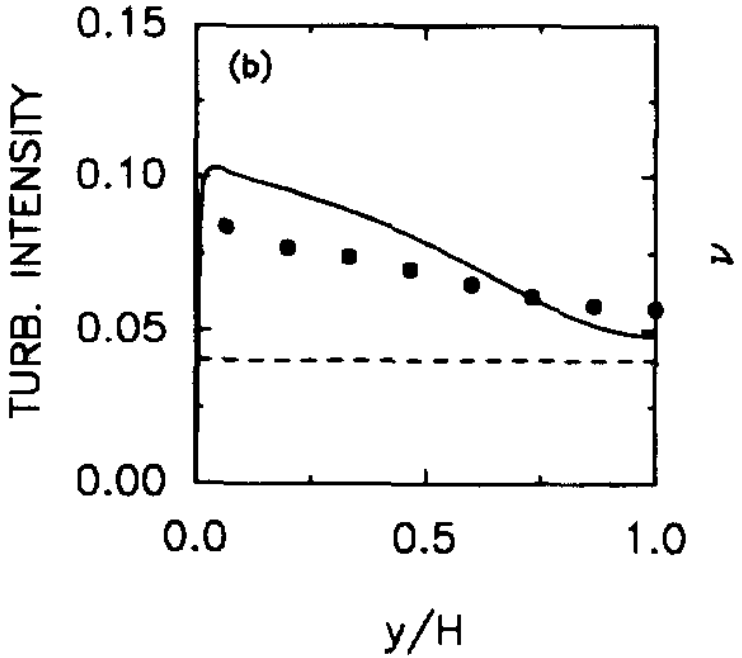
Re=2.3x10^4, d=1 mm

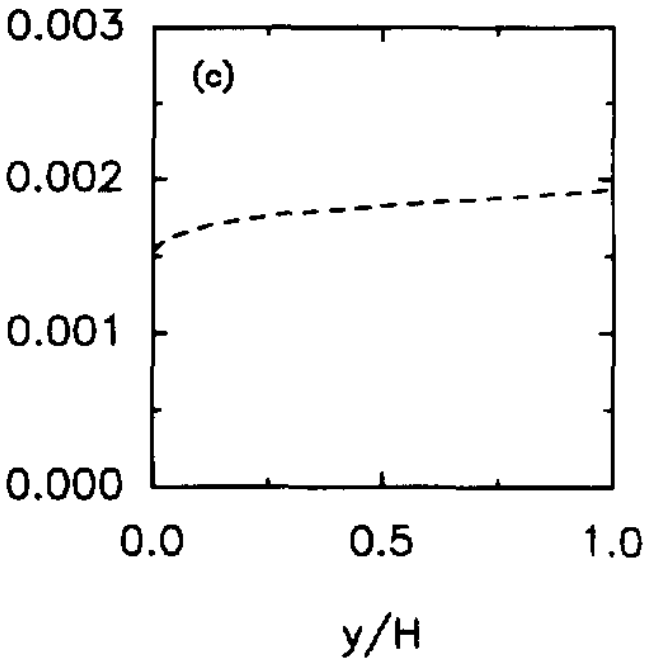
m=0.6, u_c=13.4 m/s

- - - - - Prediction (particle)

——— Prediction (air)

• Tsuji et al. (air)





Turb. Intensity vs Mass Loading

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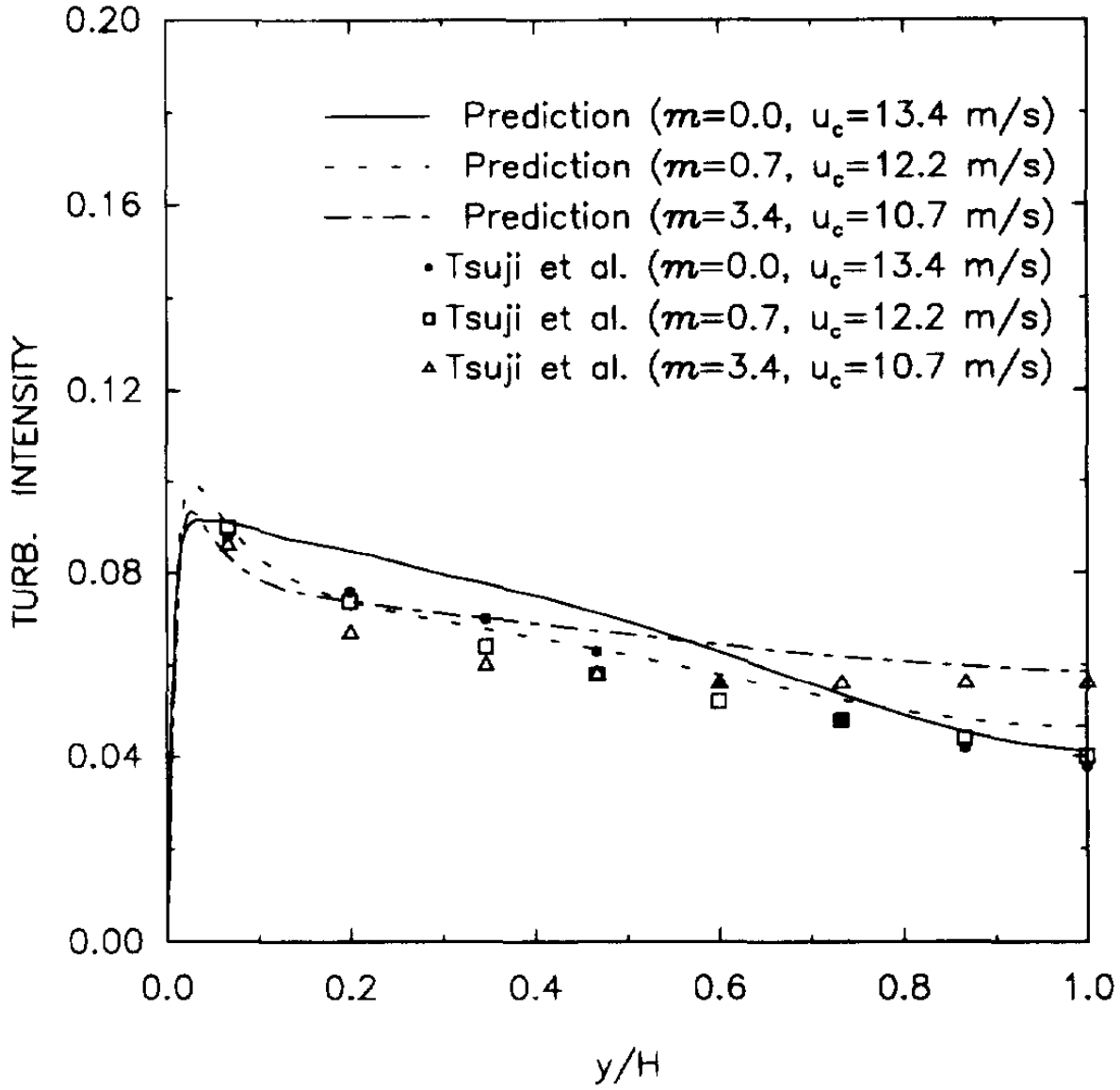
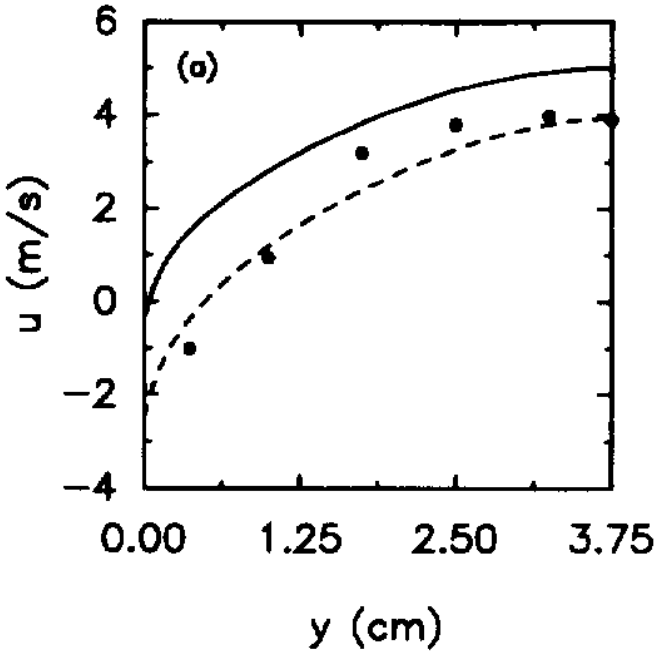


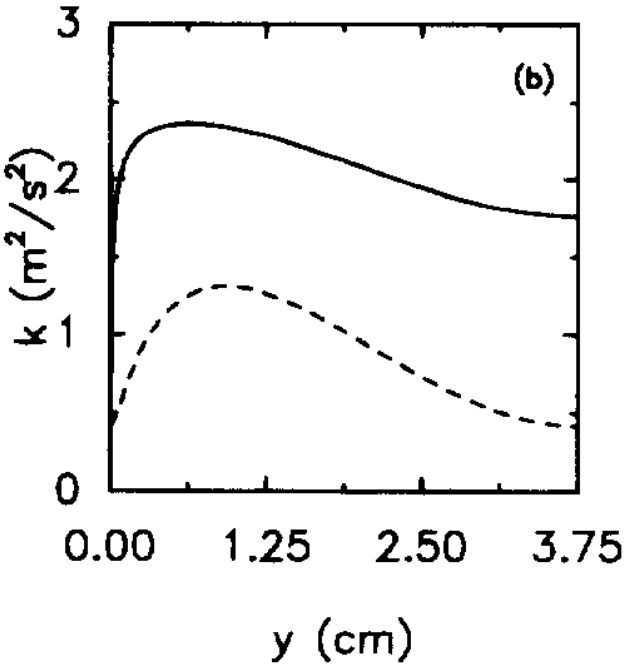


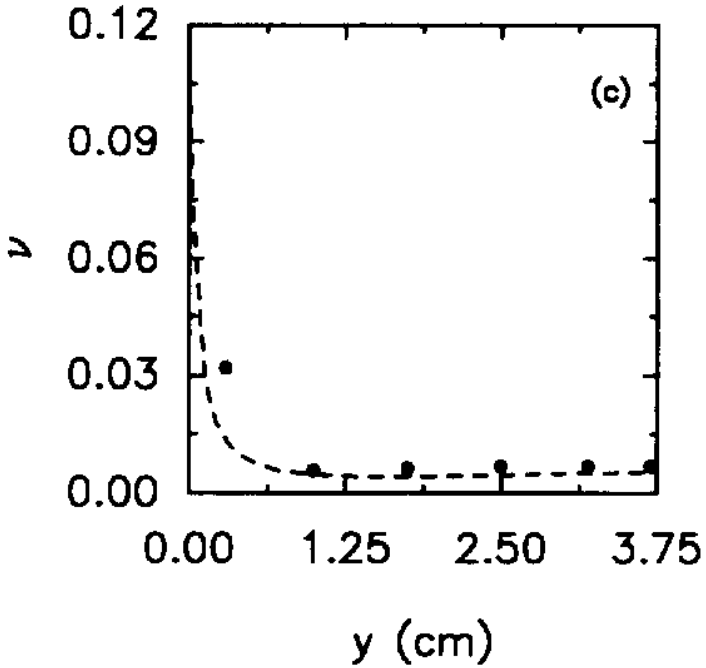
m=2.2, d=0.15 mm

Prediction (particle)

Prediction (air)

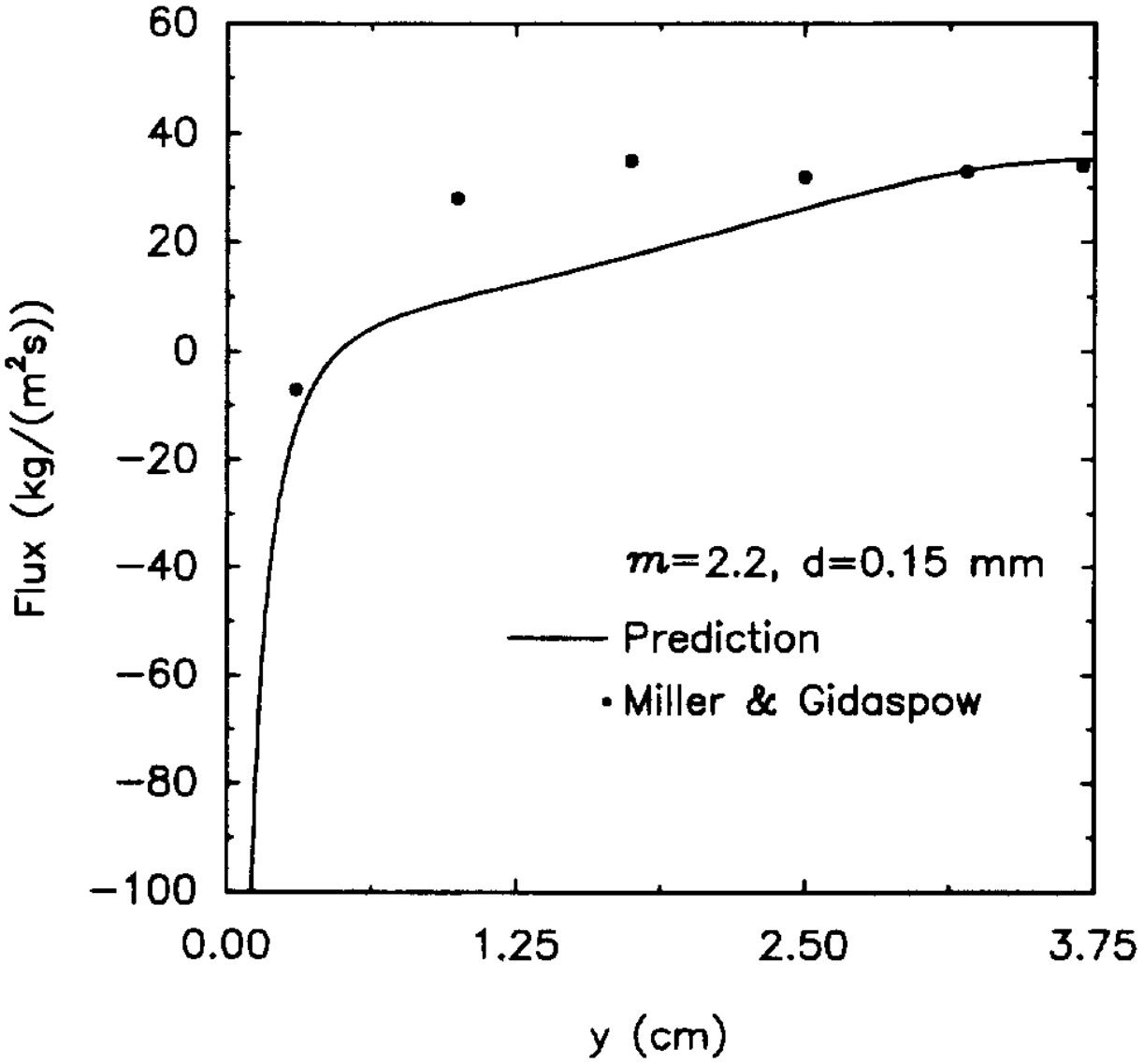
• Miller & Gidaspow (particle)

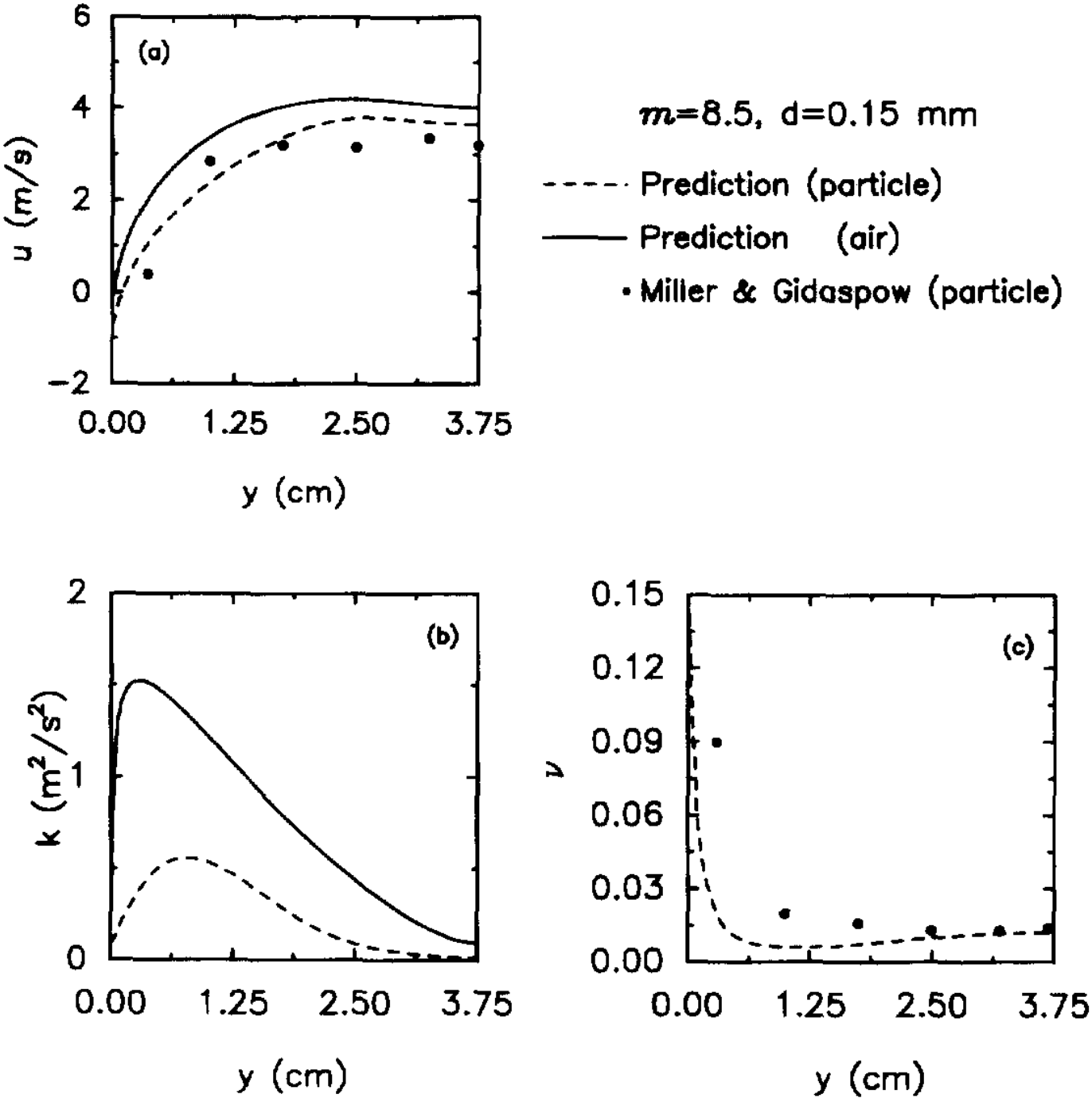




Particle Mass Flux

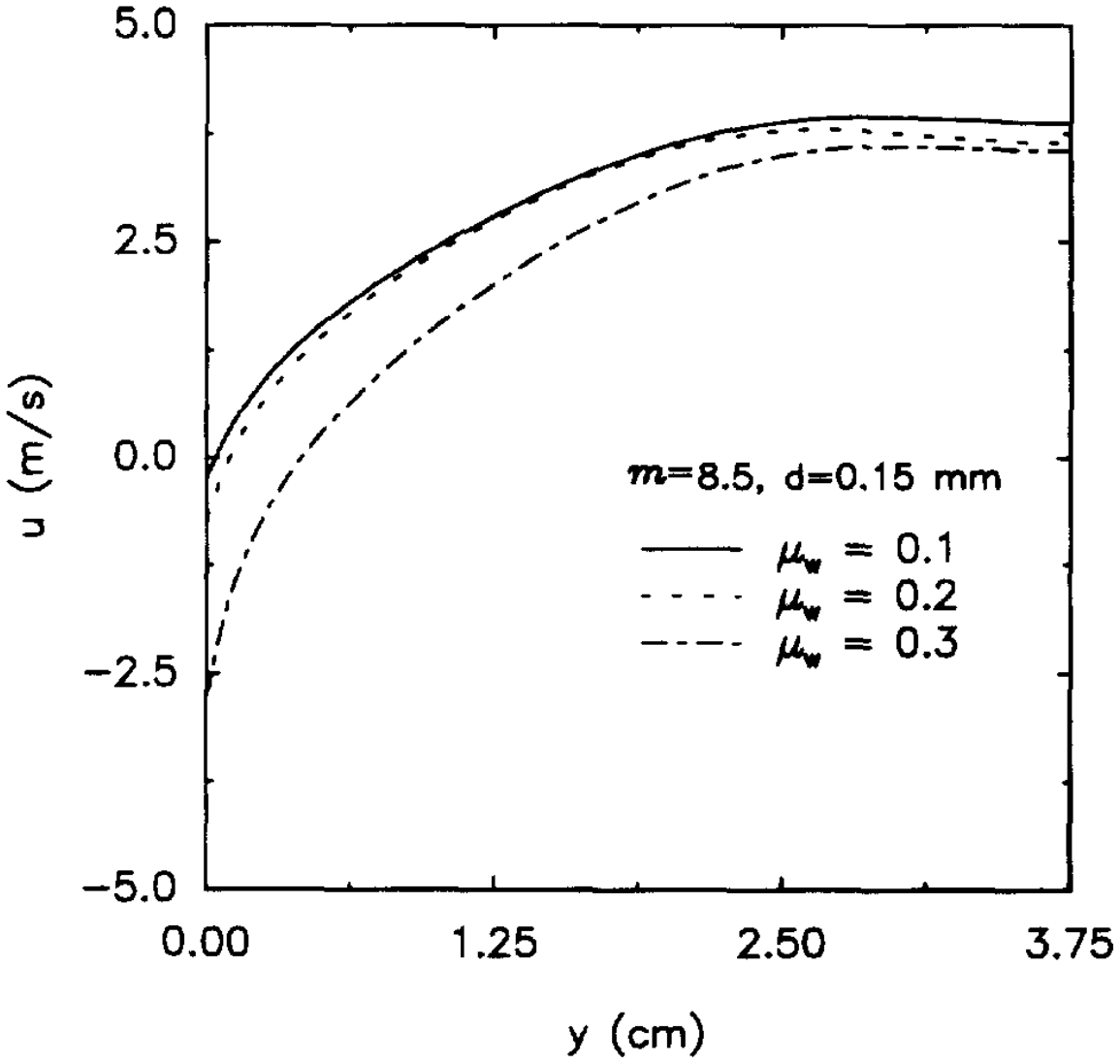
Clarkson University

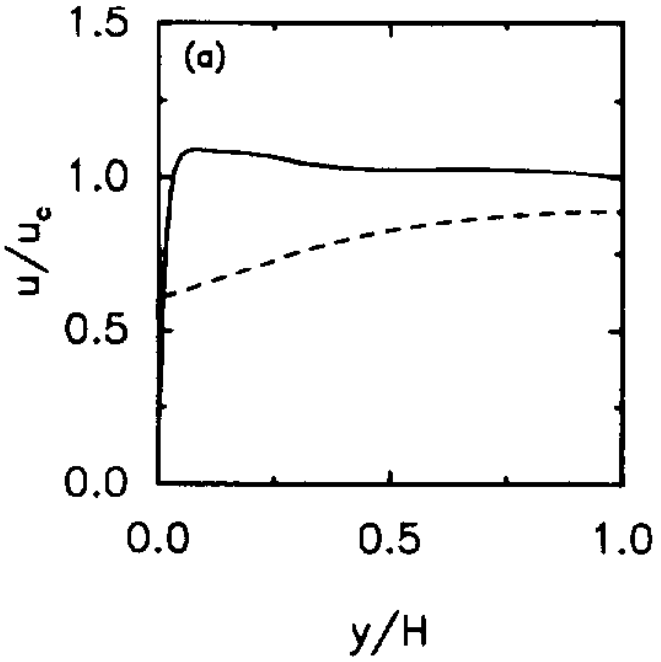



(a)
u (m/s)
6
4
2
0
−2
0.00
1.25
2.50
3.75
y (cm)
m=8.5, d=0.15 mm
Prediction (particle)
Prediction (air)
Miller & Gidaspow (particle)
(b)
k (m²/s²)
2
1
0
0.00
1.25
2.50
3.75
y (cm)
(c)
ν
0.15
0.12
0.09
0.06
0.03
0.00
0.00
1.25
2.50
3.75
y (cm)

Effect of Wall Friction

Clarkson University





Re=1.2x10^4, d=0.5 mm

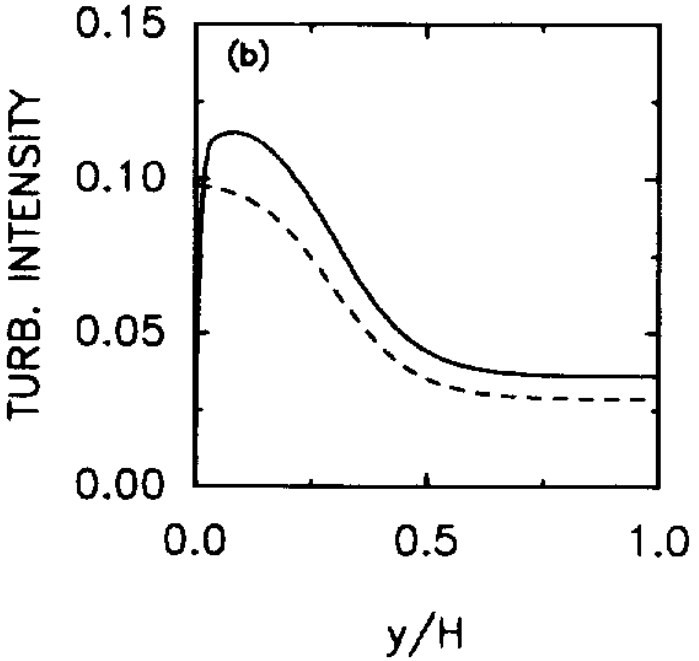
m=216, u_c=5.76 m/s

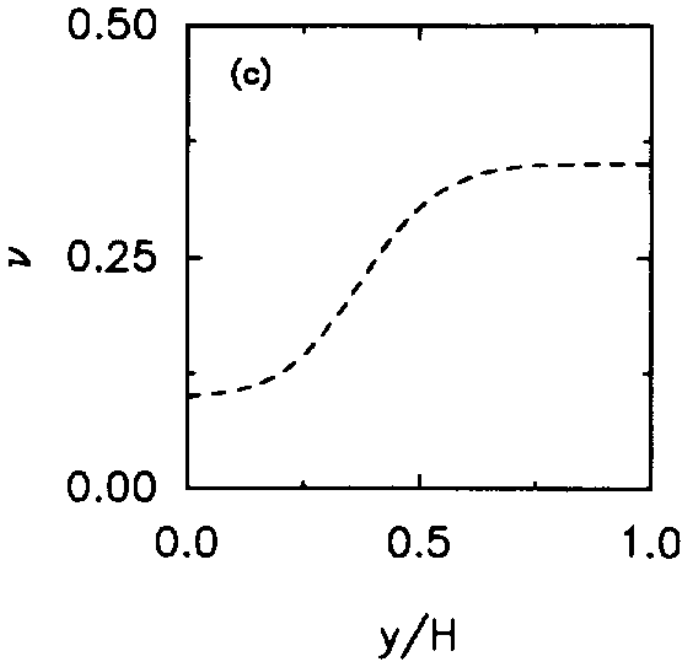
Prediction (particle)

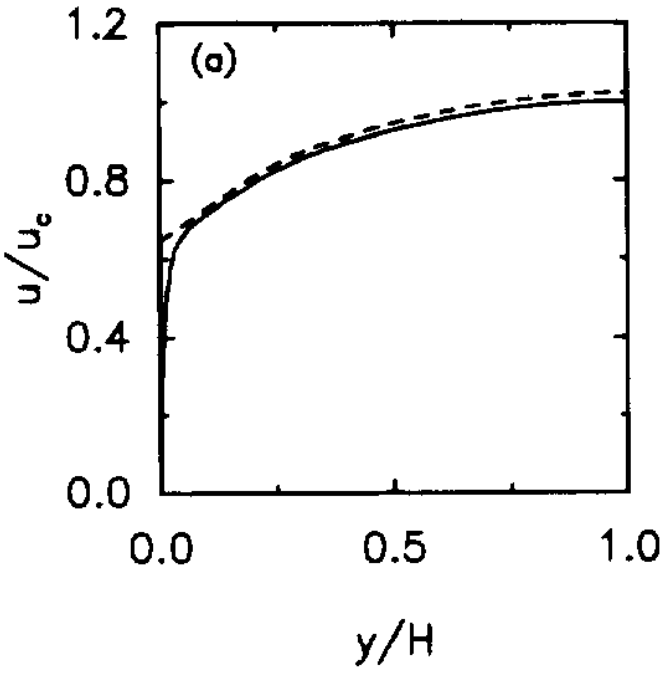
Prediction (air)

Clarkson University

Upward Flow







Re=1.0x10^4, d=0.5 mm

m=646, u_c=5.18 m/s

----- Prediction (particle)

——— Prediction (air)

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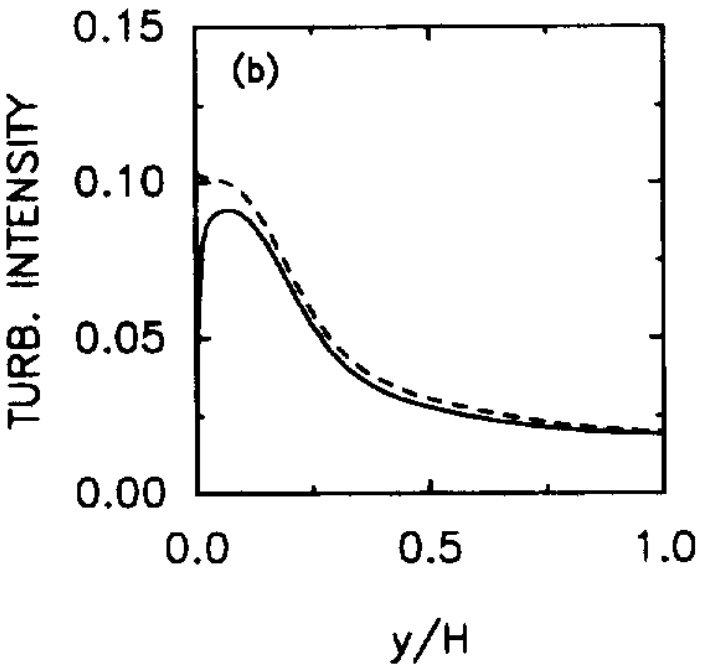
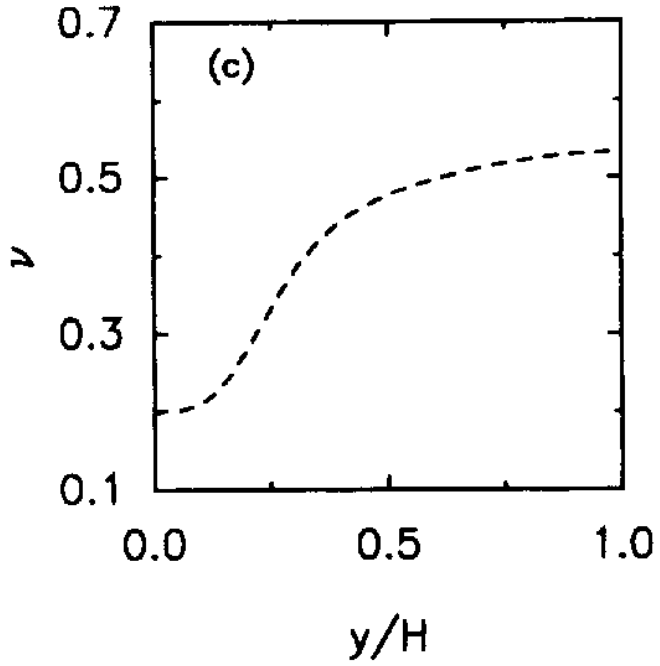
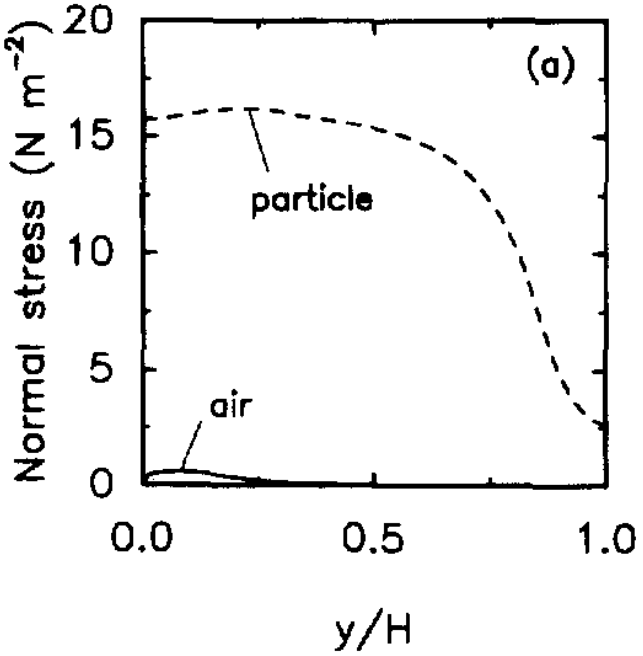
Downward Flow







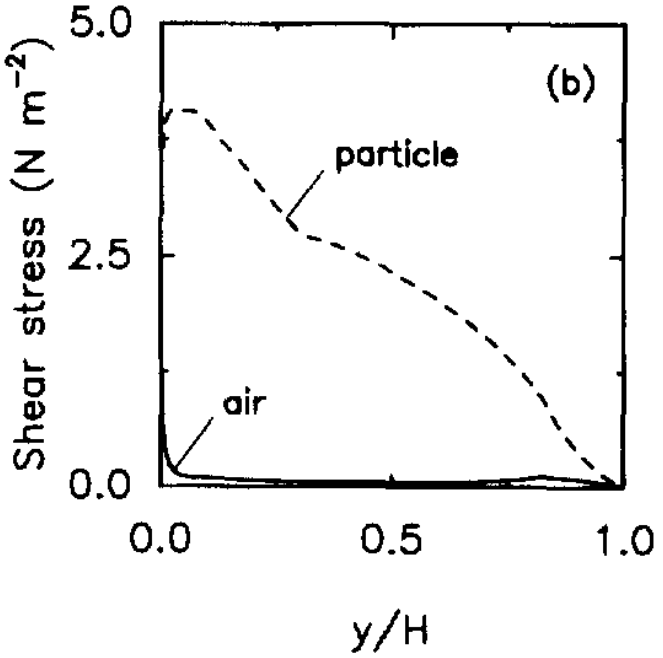
m=646

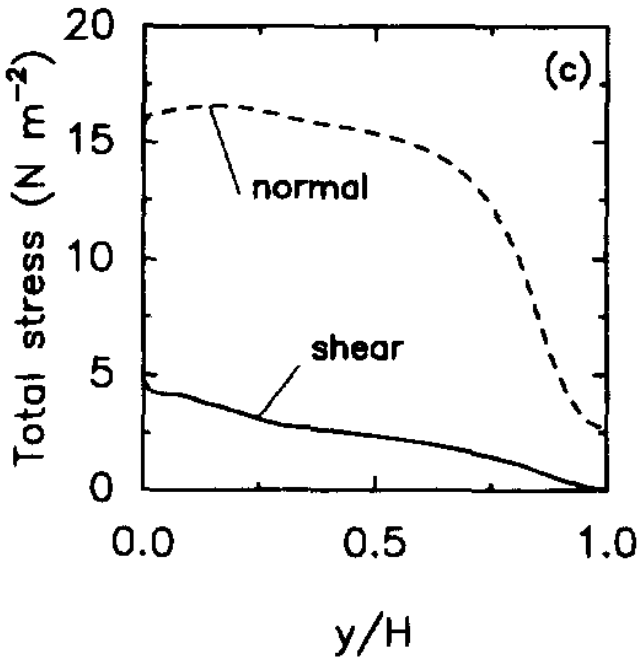
u_c=5.18 m/s

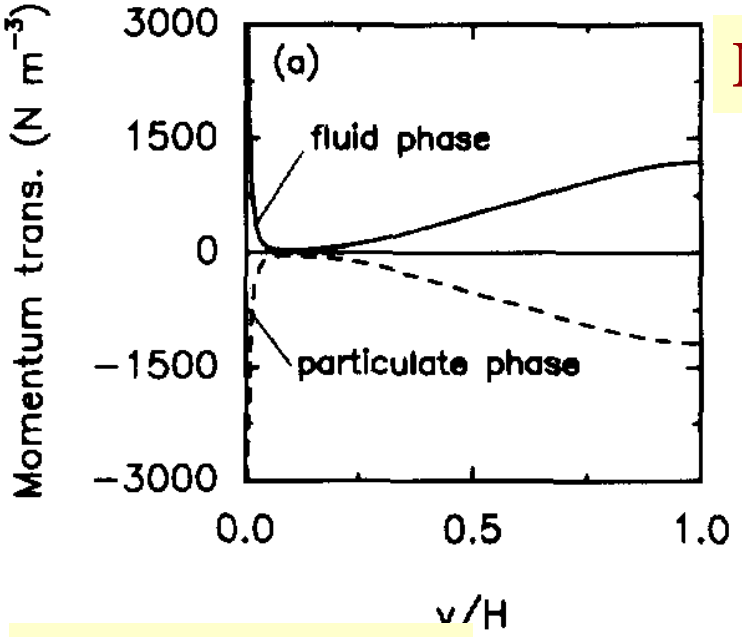
Re=1.0x10^4, d=0.5 mm

Clarkson University

Downward Flow







Momentum Supply

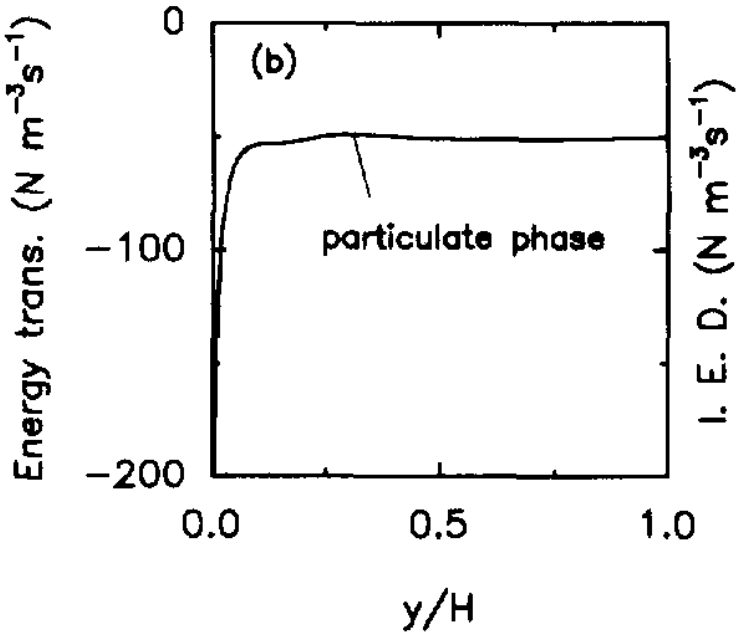
m=646

u =5.18 m/s

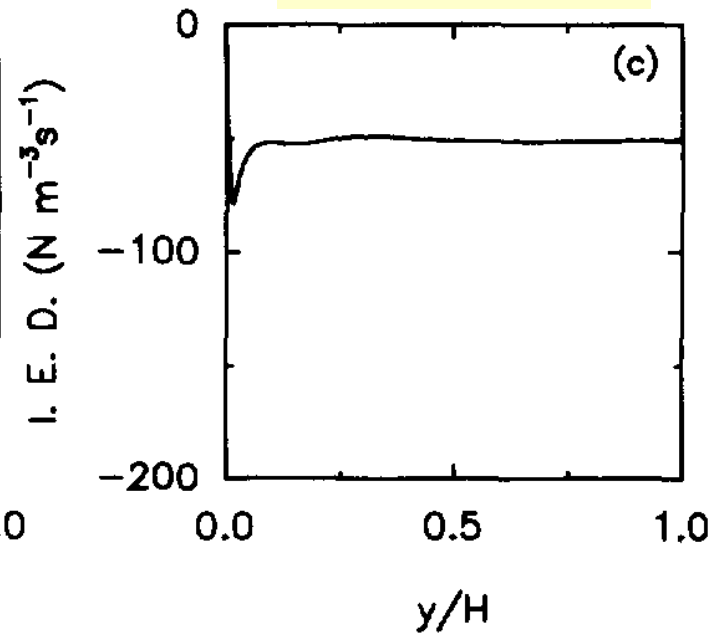
Re=1.0x10^4, d=0.5 mm

Downward Flow

Energy Supply



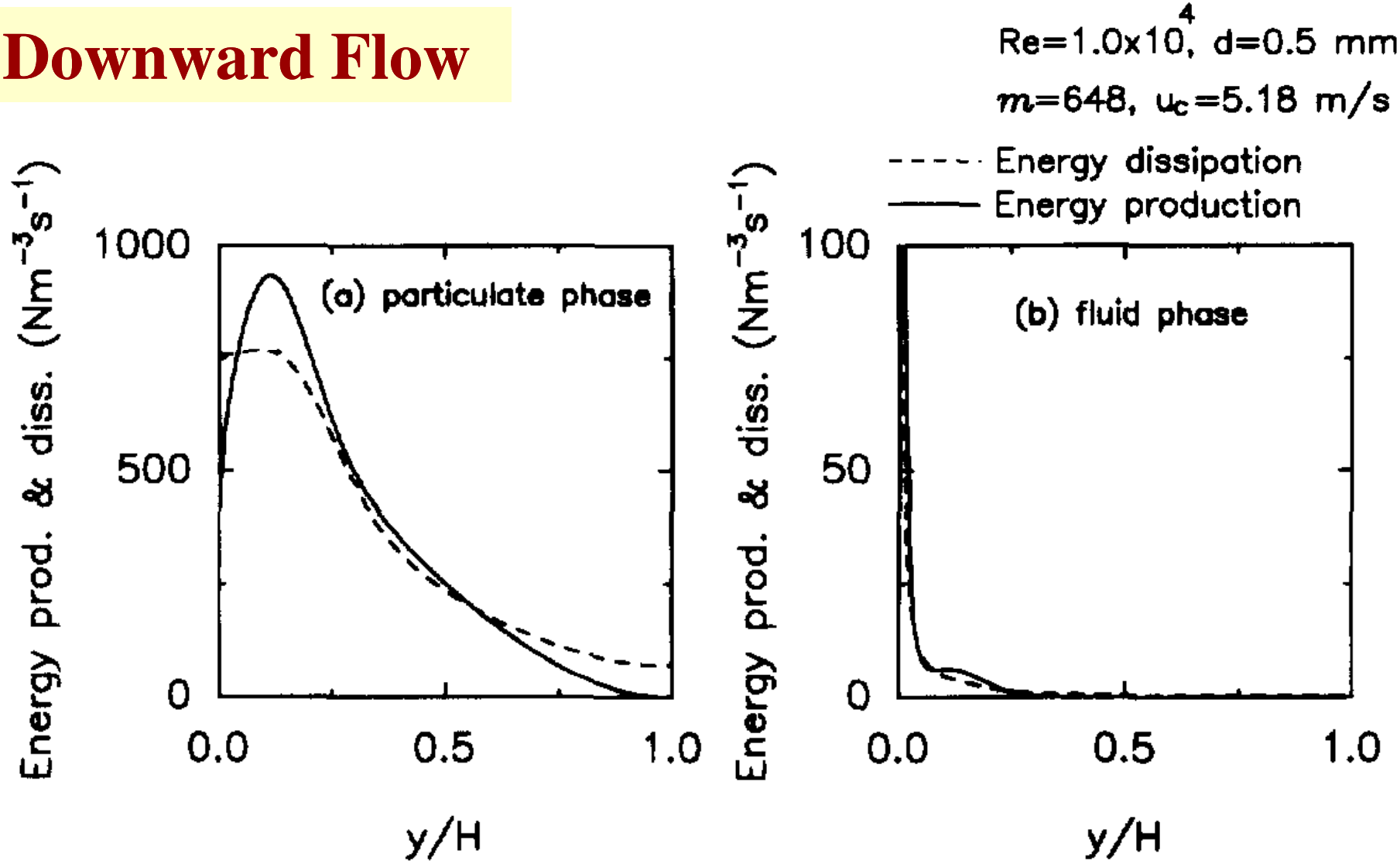
Interaction Dissipation



Energy Production/ Dissipation

Clarkson University

Downward Flow



Conclusions

Clarkson University

- Fluctuation energies of gas and particles are important in dynamic of two phase flows.
- Interaction momentum and energy supplies are both important.
- For mass loading of 0.6 or larger, the production of particulate fluctuation energy is important and exceeds its direct dissipation.
- The model predictions agreed well with the experimental data.

Particuology 10 (2012) 553–561

Contents lists available at SciVerse ScienceDirect

Particuology

journal homepage: www.elsevier.com/locate/partic

Numerical evaluation of turbulence models for dense to dilute gas–solid flows in vertical conveyor

Salar Azizi[a], Dariush Mowla[a,*], Goodarz Ahmadi[b]

Matsumoto and Harakava (1987) studied experimentally the flow in a pneumatic conveyer which was 10 cm wide and 5.6 m long.

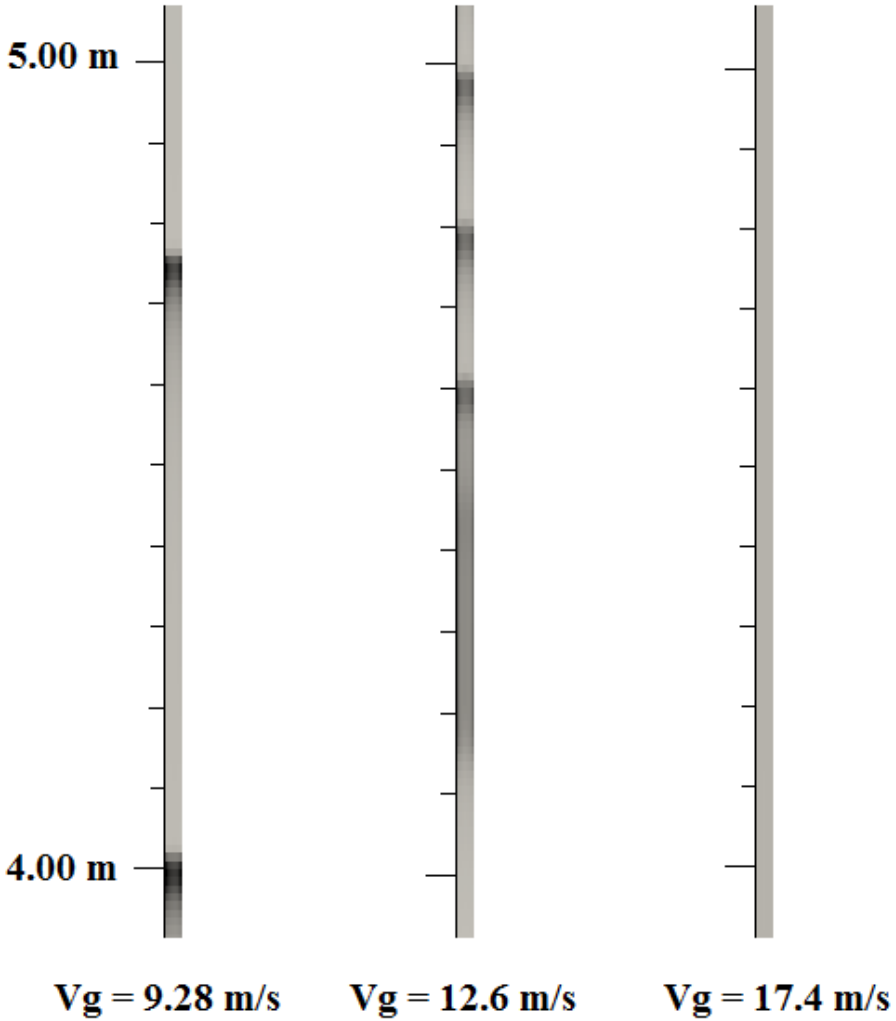
Properties and Parameters

Clarkson University

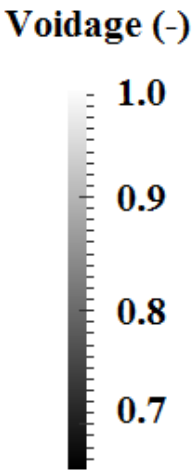
	Property	Unit	Value
d_p	Particle diameter	mm	1.91
ρ_s	Solids density	kg/m^3	2500
e	Restitution	-	0.9
e_w	Wall restitution	-	0.2 to 0.7
φ	Specularity	-	0.008 to 0.028
$\alpha_{s\text{-max}}$	Max solids volume	-	0.55
ρ_g	Inlet gas density	kg/m^3	1.185
μ_g	Inlet gas viscosity	Pas	1.14×10^{-5}
V_g^{inlet}	Superficial velocity	m/s	9.28, 10.3, 12.6, 14.4, 17.4
M_s	Solid mass flow rate	Kg/s	0.020

Voidage Contours – Ahmadi-Ma Model

Clarkson University



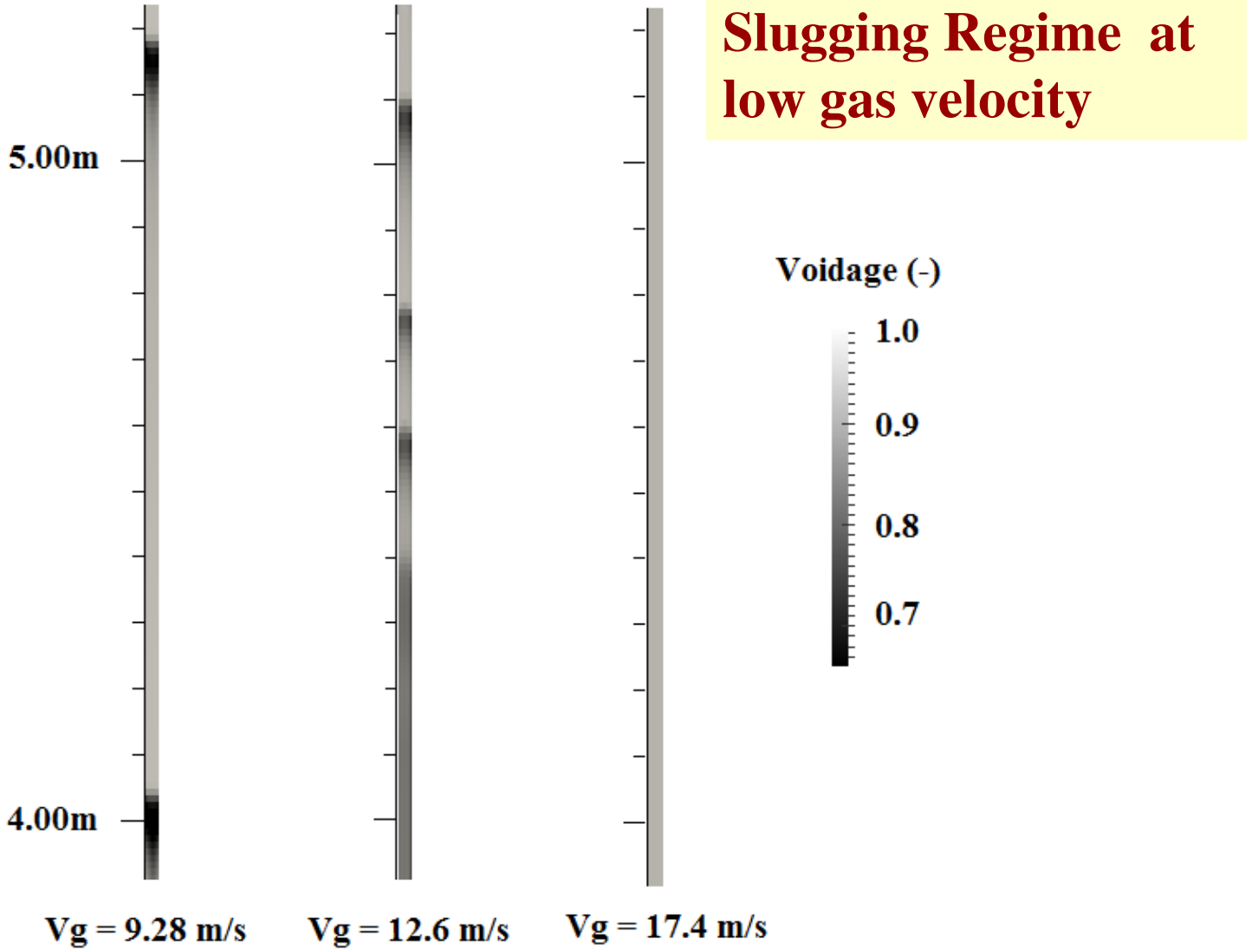
Slugging Regime at low gas velocity



Azizi, Mowla, Ahmadi (2012)

Voidage Contours – Simonin Model

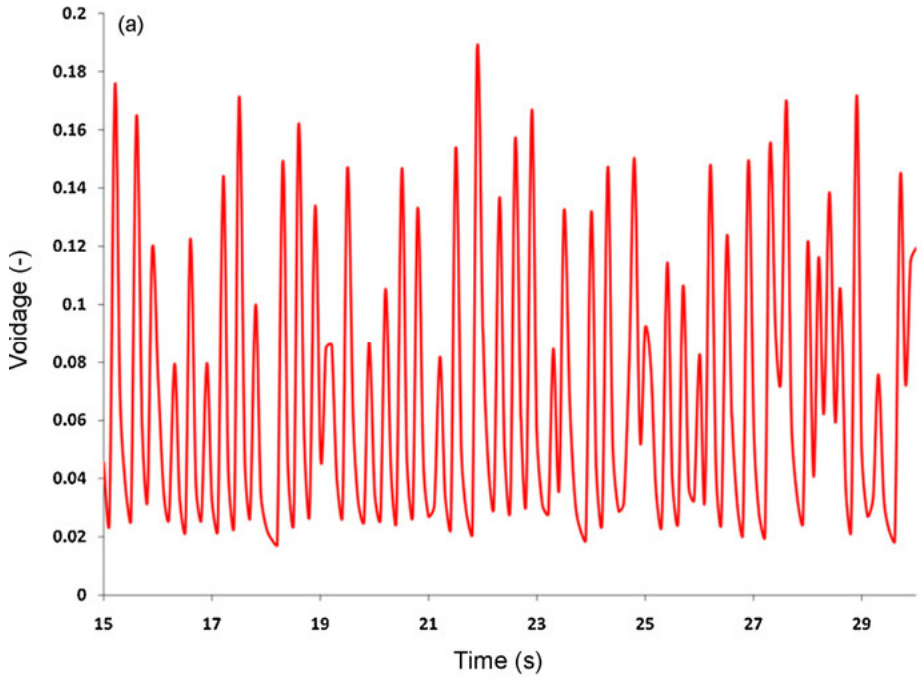
Clarkson University



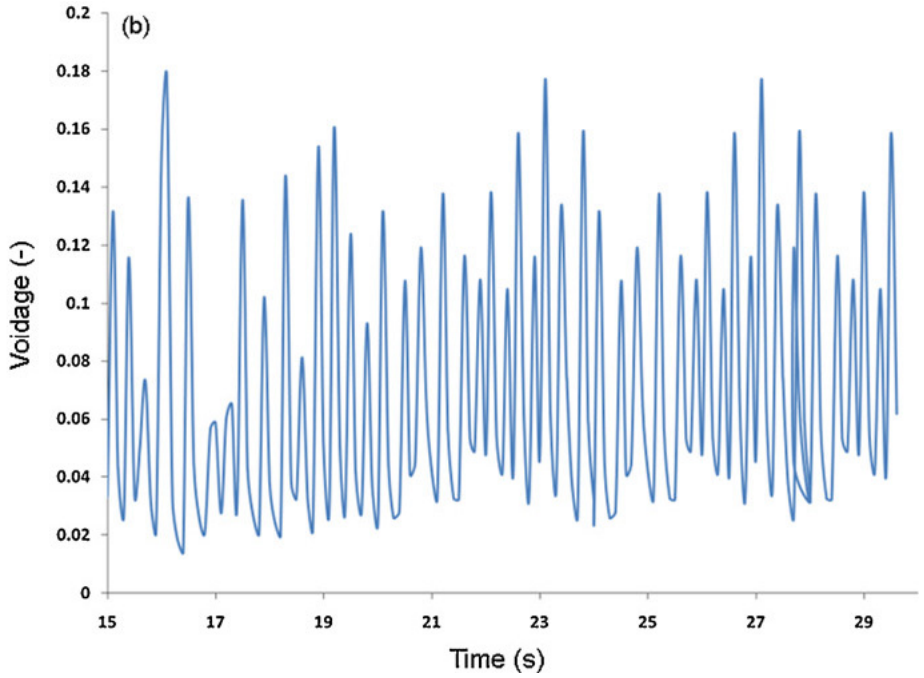
Slugging Regime at low gas velocity

Outlet Voidage Fluctuation

Clarkson University



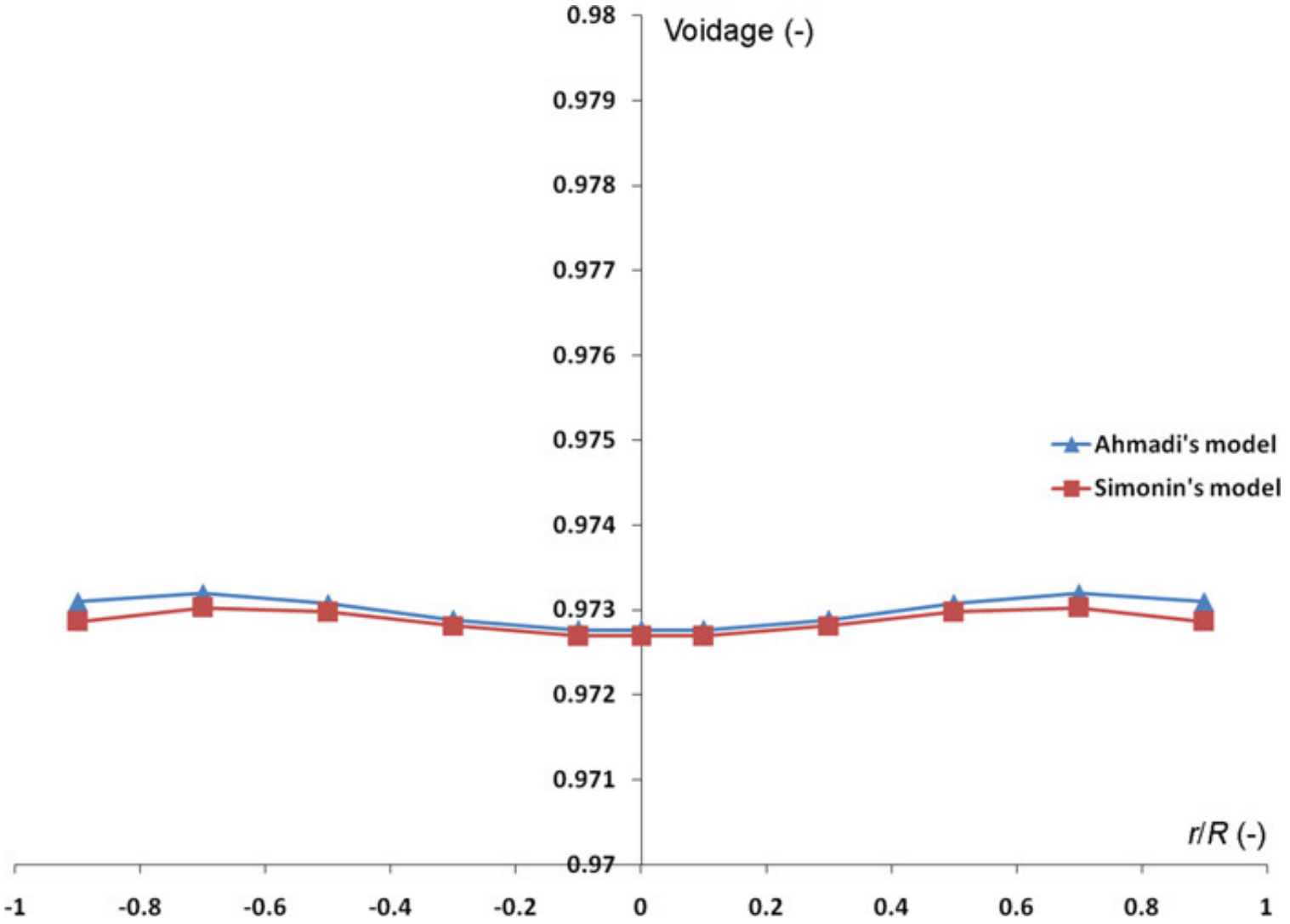
Simonin



Ahmadi-Ma

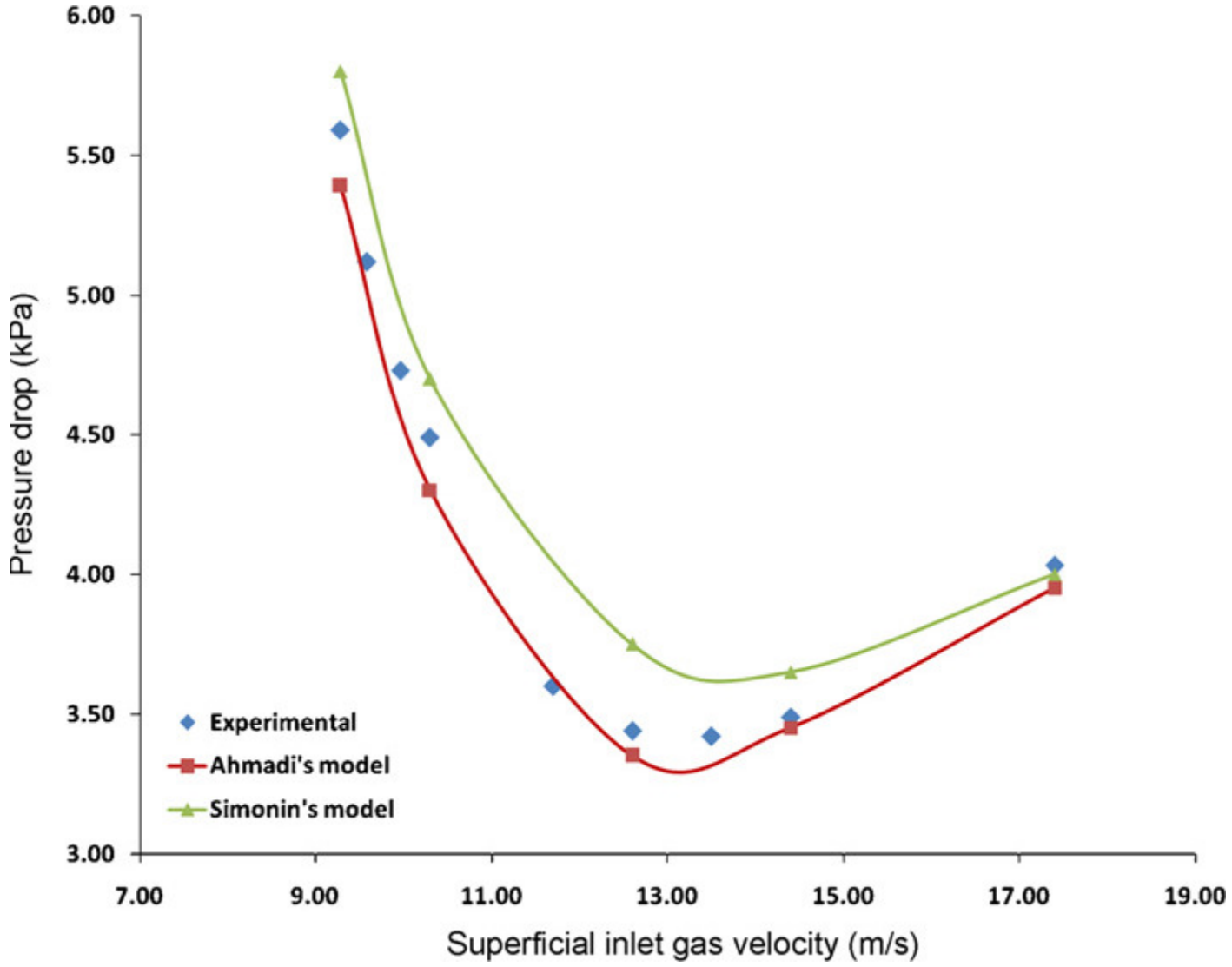
Voidage Profile

Clarkson University



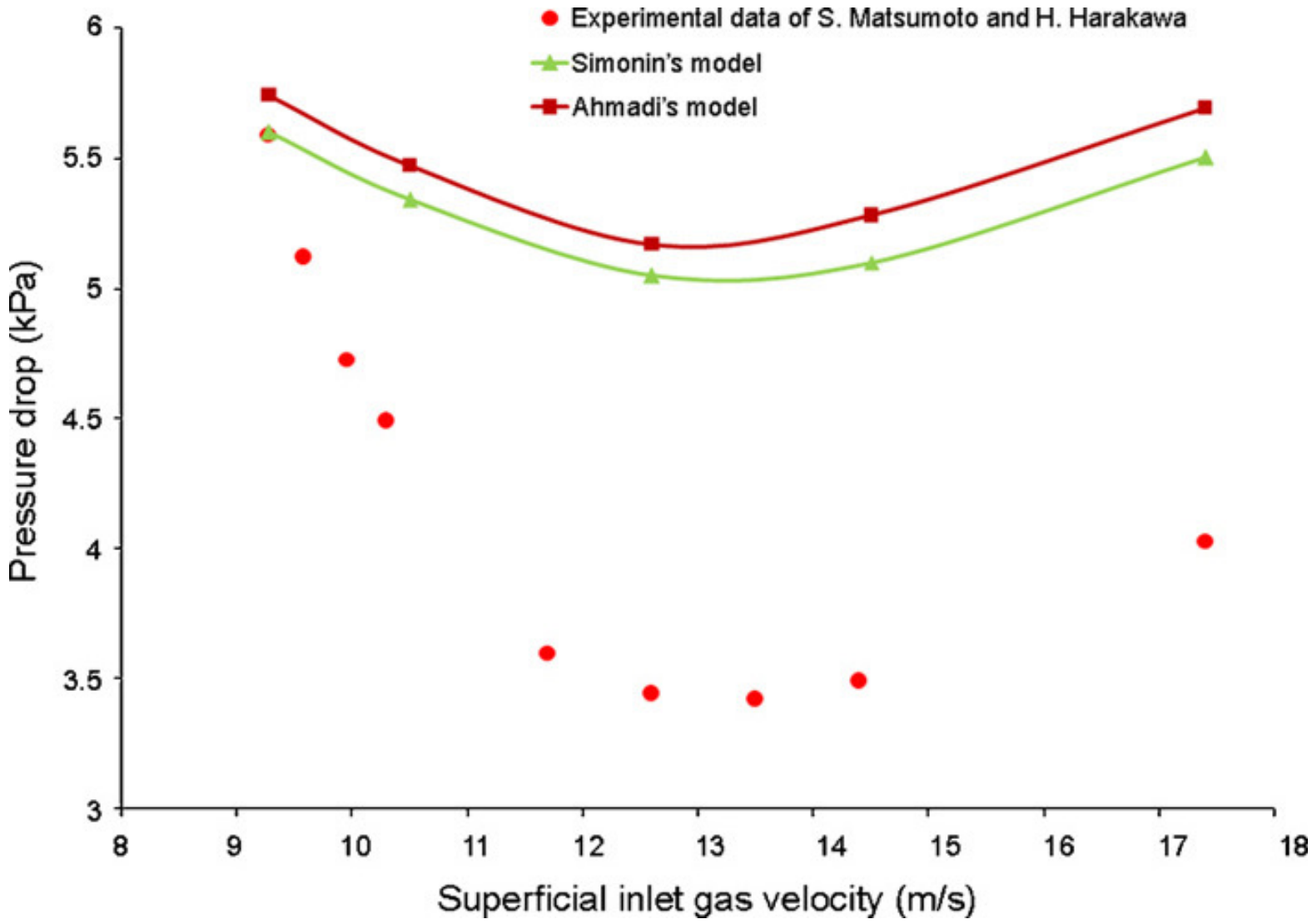
Pressure Drop

Clarkson University



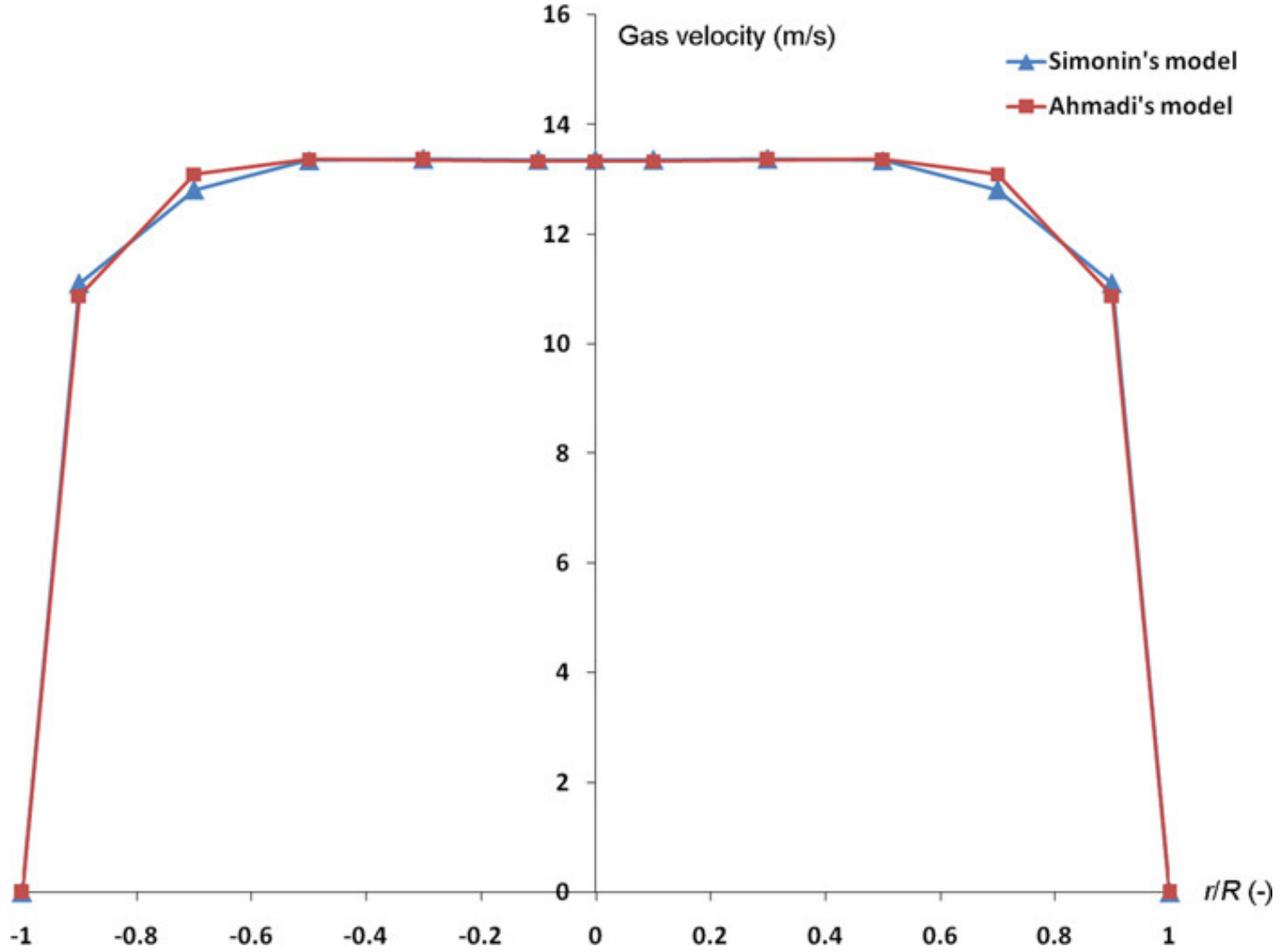
Gas Velocity Profile

Clarkson University



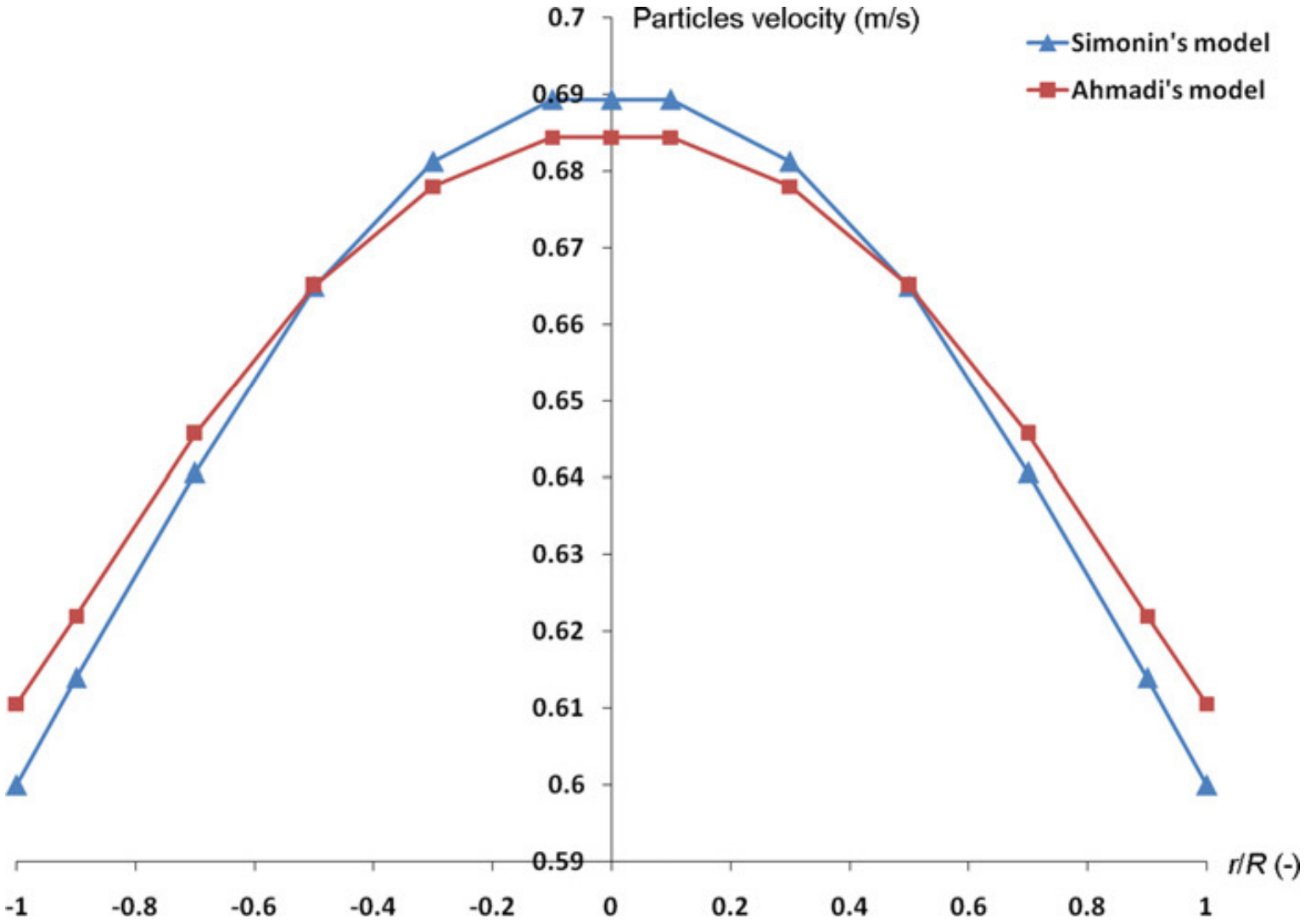
Gas Velocity Profile

Clarkson University



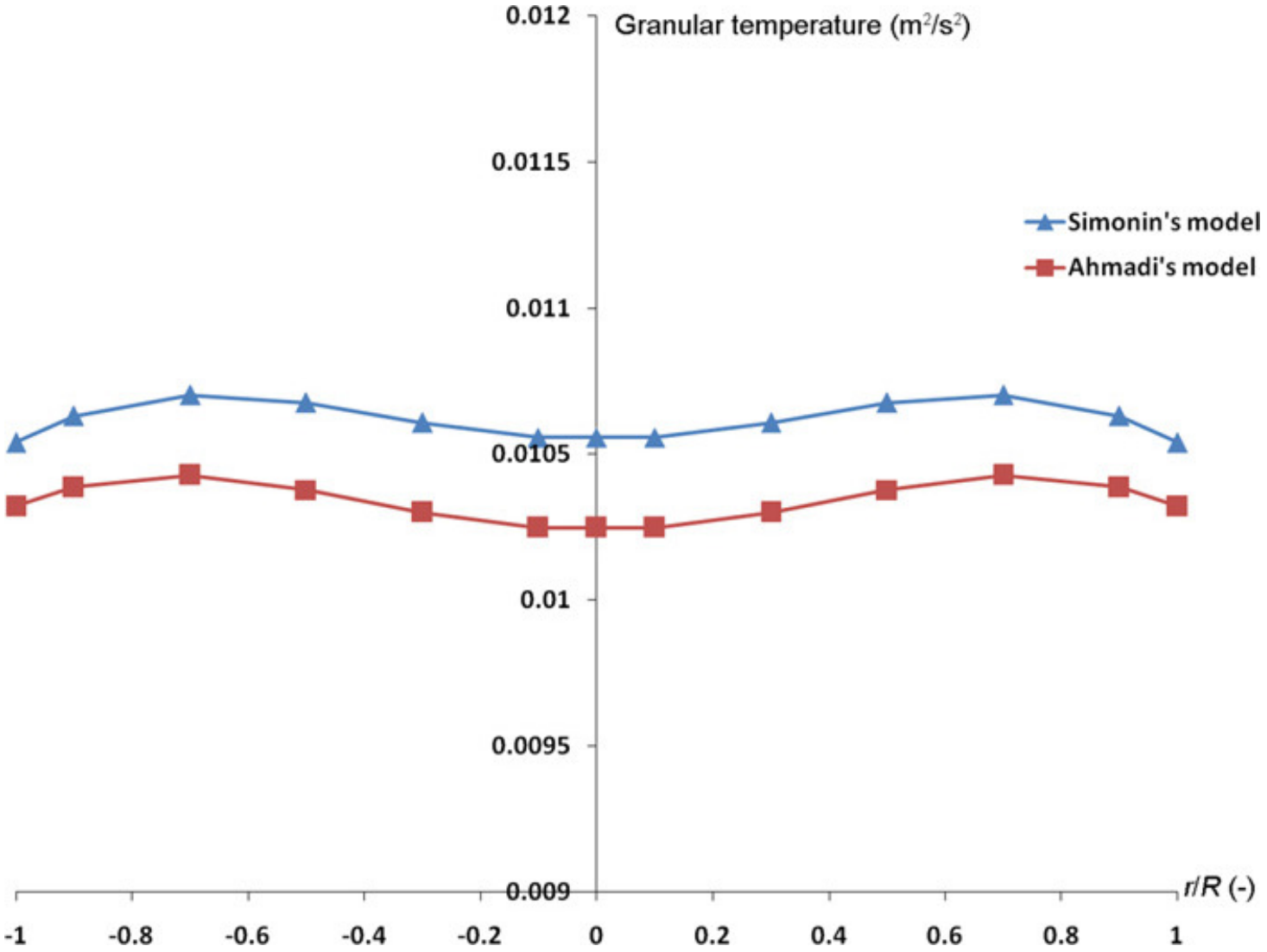
Particle Velocity Profile

Clarkson University



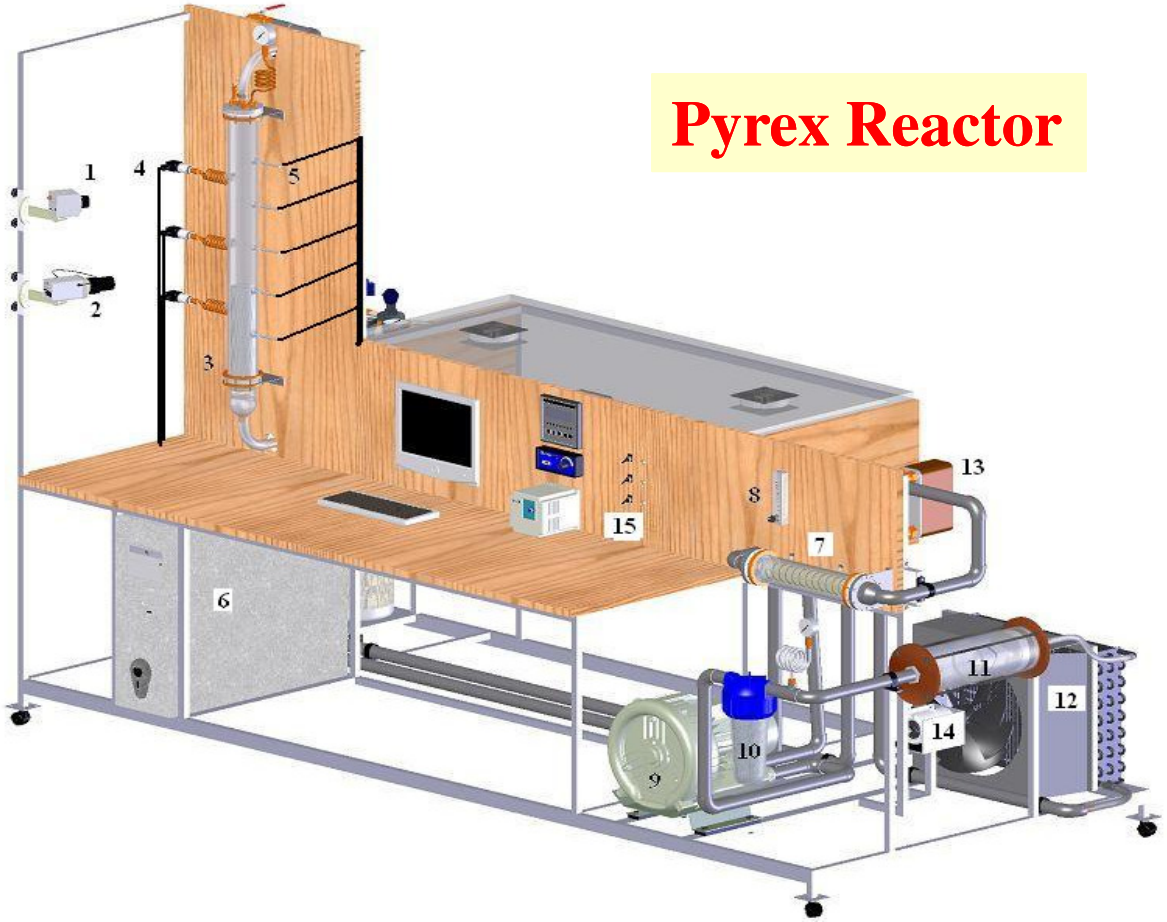
Granular Temperature Profile

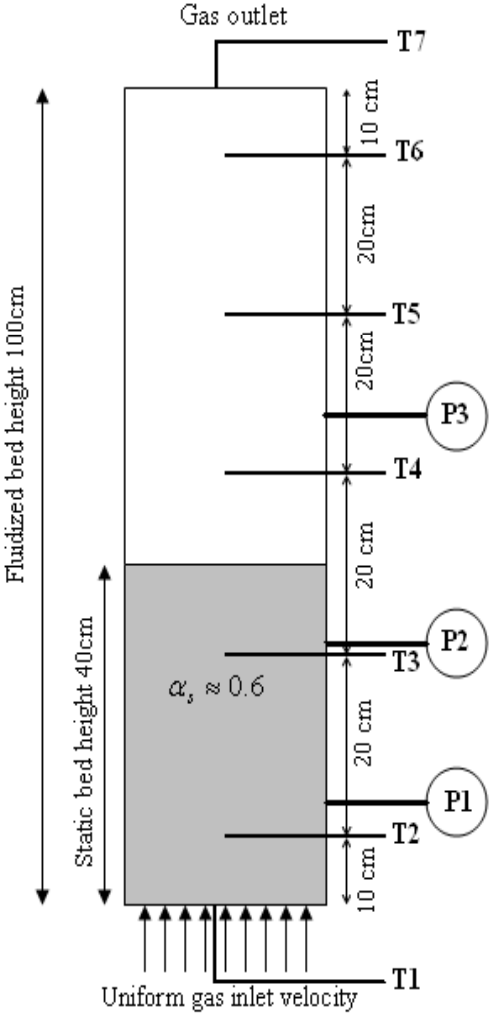
Clarkson University



Computational and Experimental Study of Heat Transfer and Hydrodynamics in a 2D Gas−Solid Fluidized Bed Reactor

Mahdi Hamzehei,[†] Hassan Rahimzadeh,[†] and Goodarz Ahmadi*[,‡]

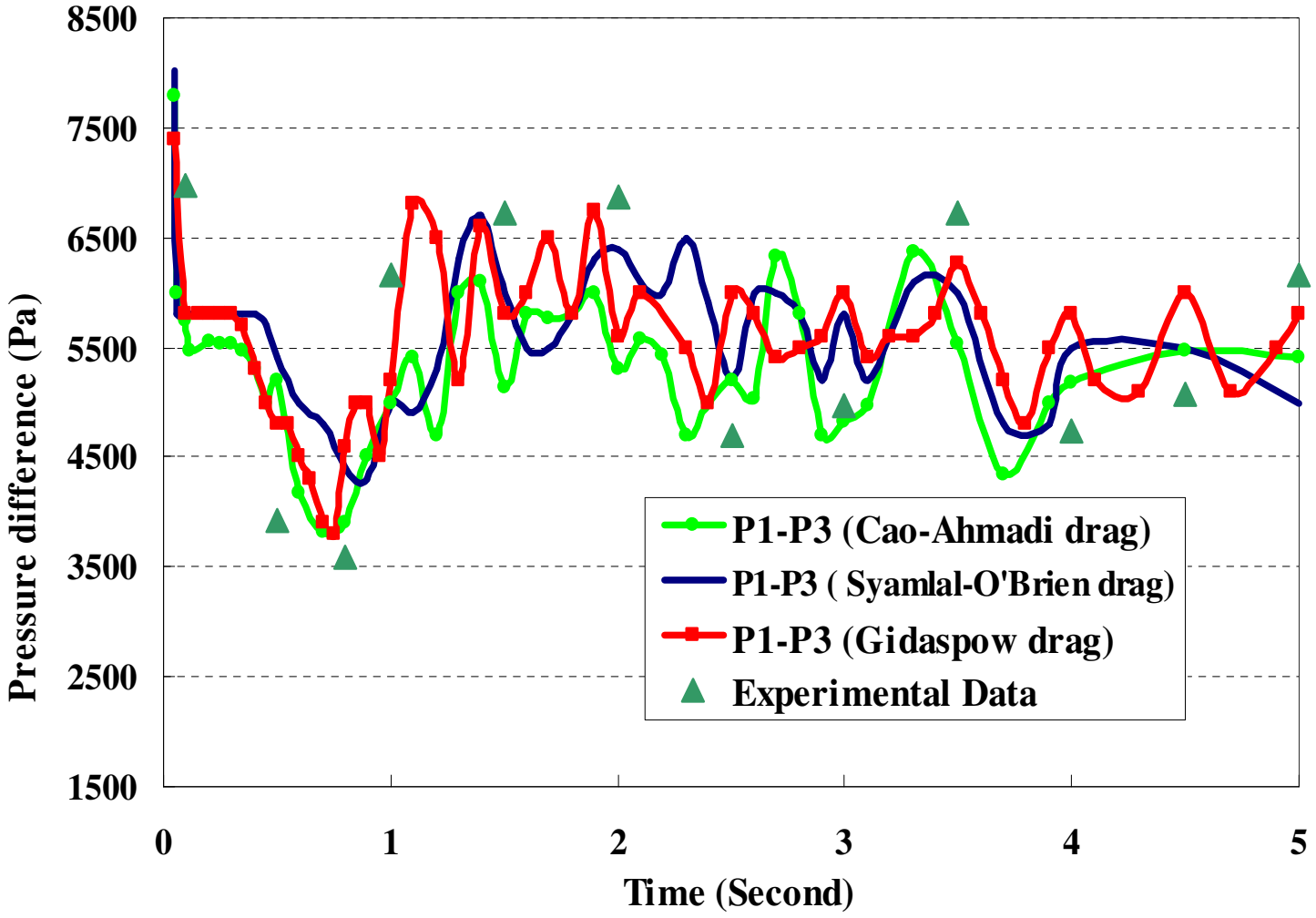




Pressure Drops

Clarkson University

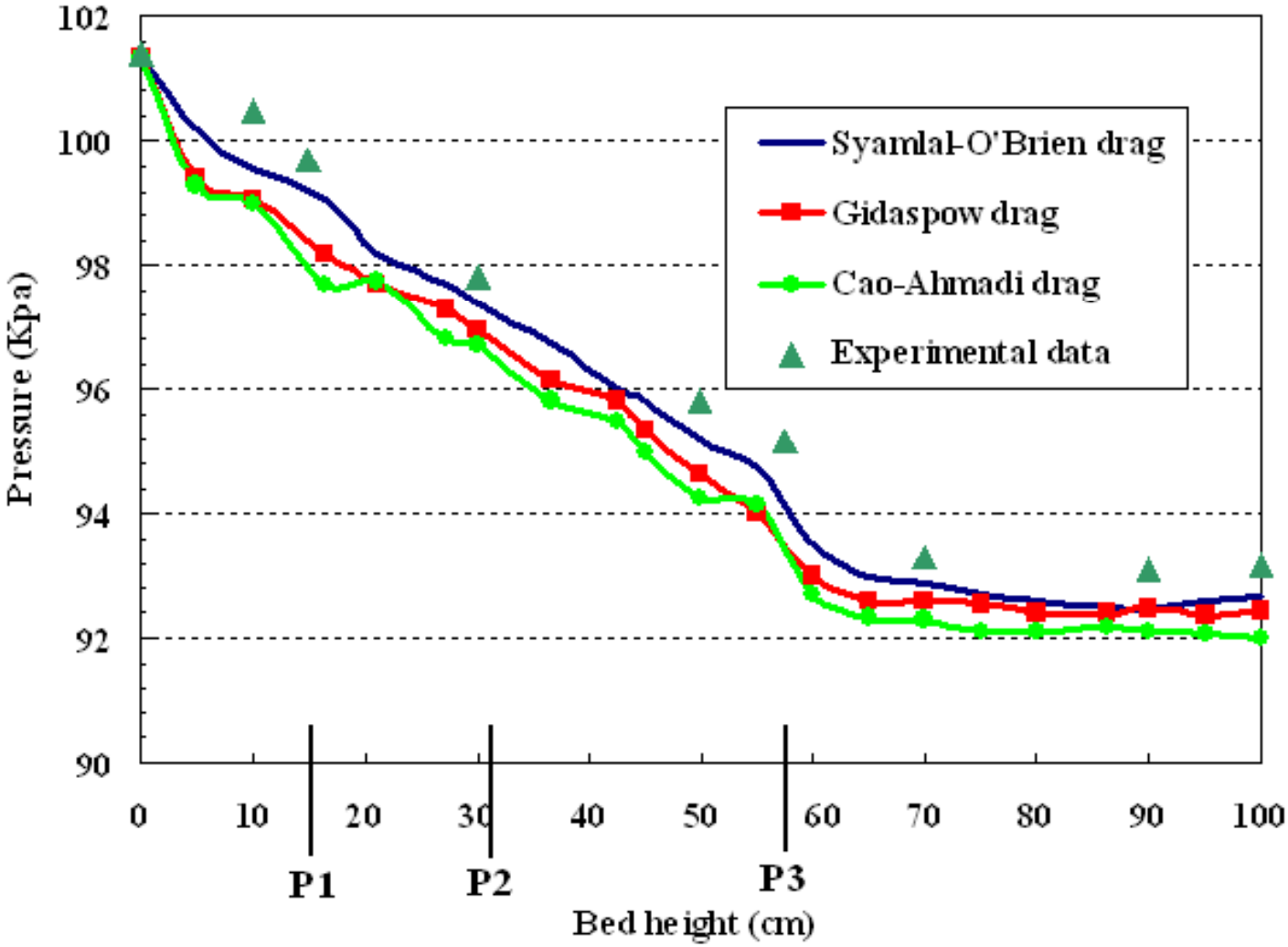
Superficial velocity V_g = 50 cm/s



Pressure Variations

Clarkson University

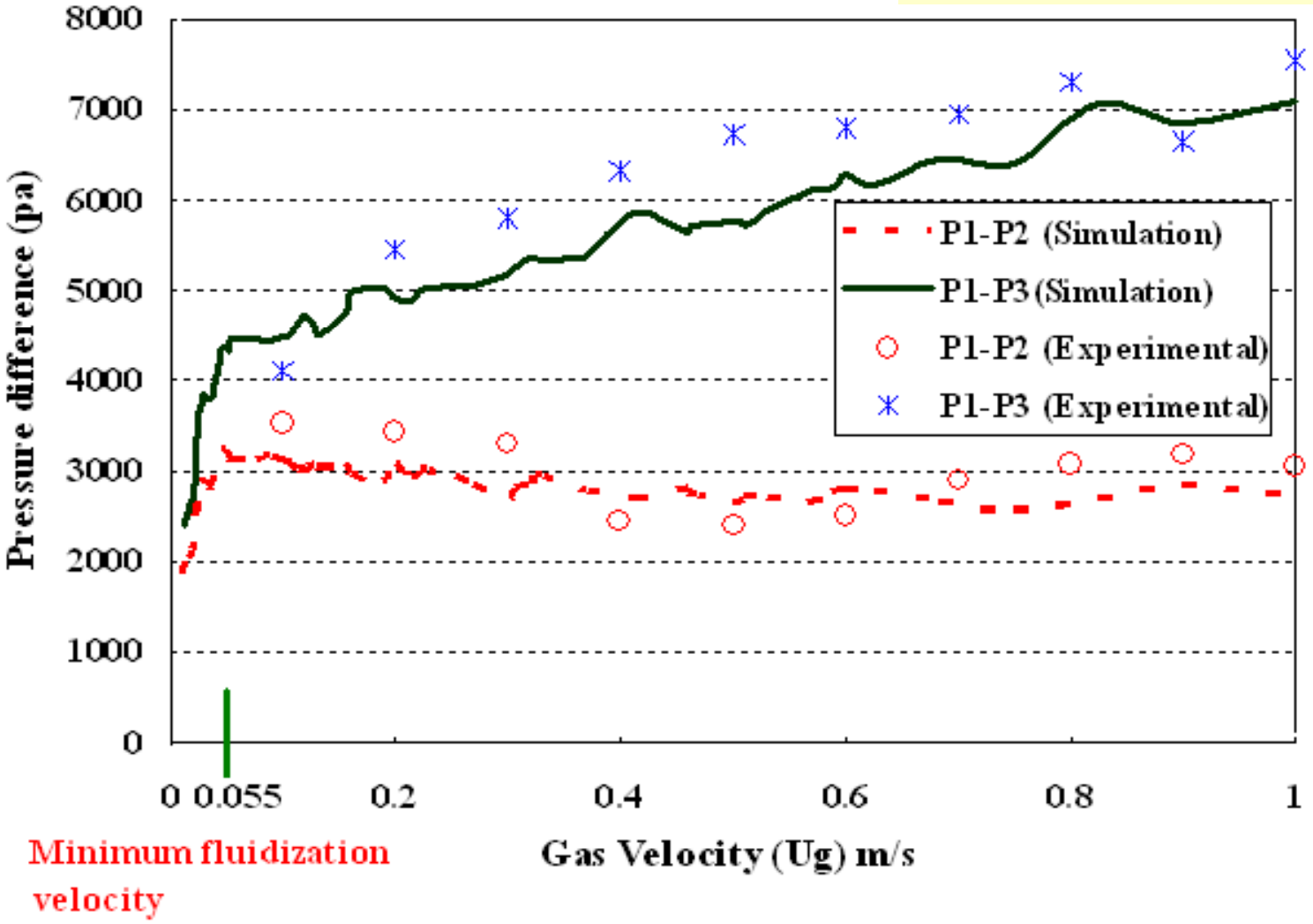
Superficial velocity V_g = 50 cm/s



Pressure Drops vs V_g

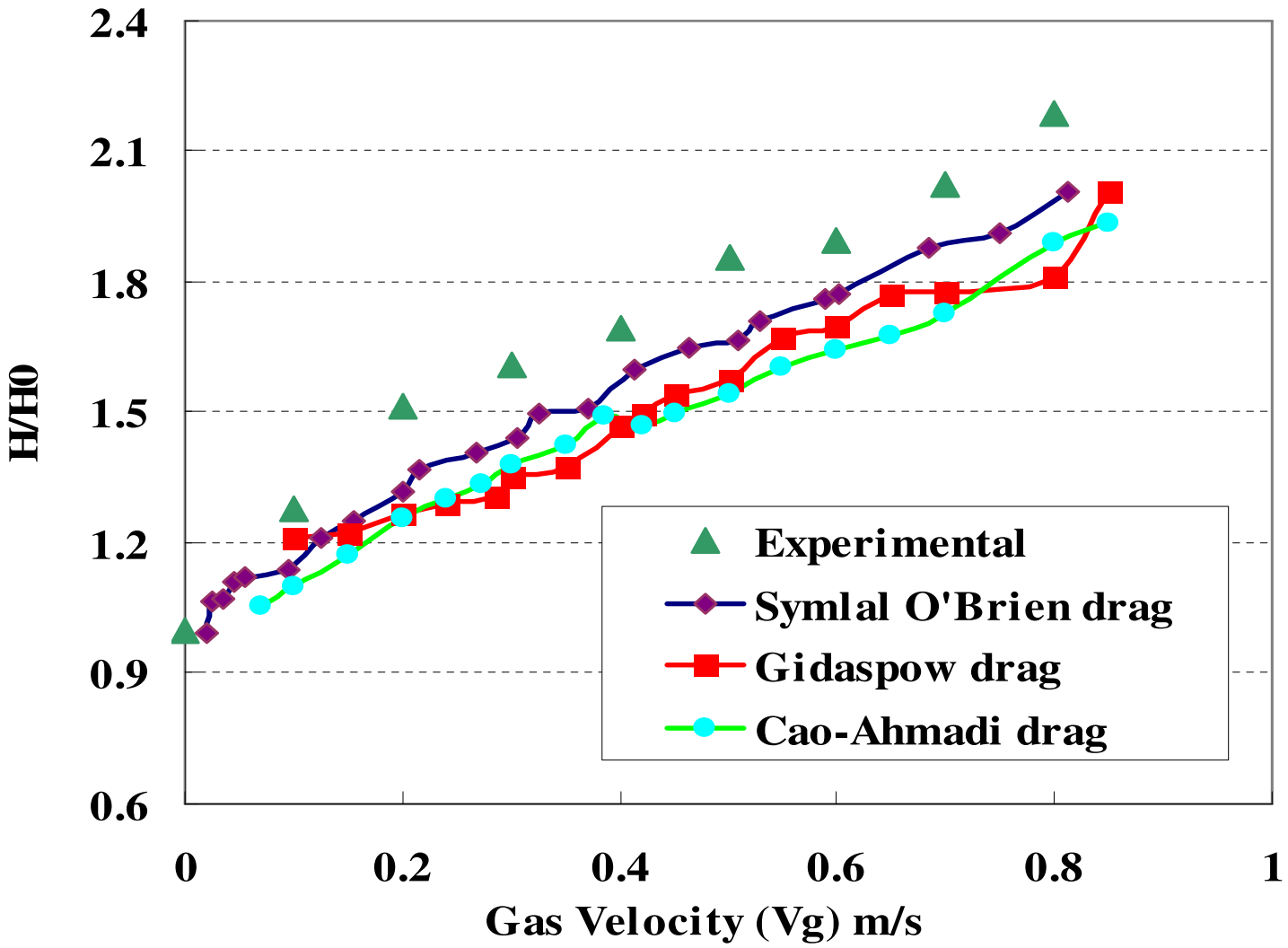
Clarkson University

Syamlal–O'Brien Drag



Bed Expansion Ratio

Clarkson University

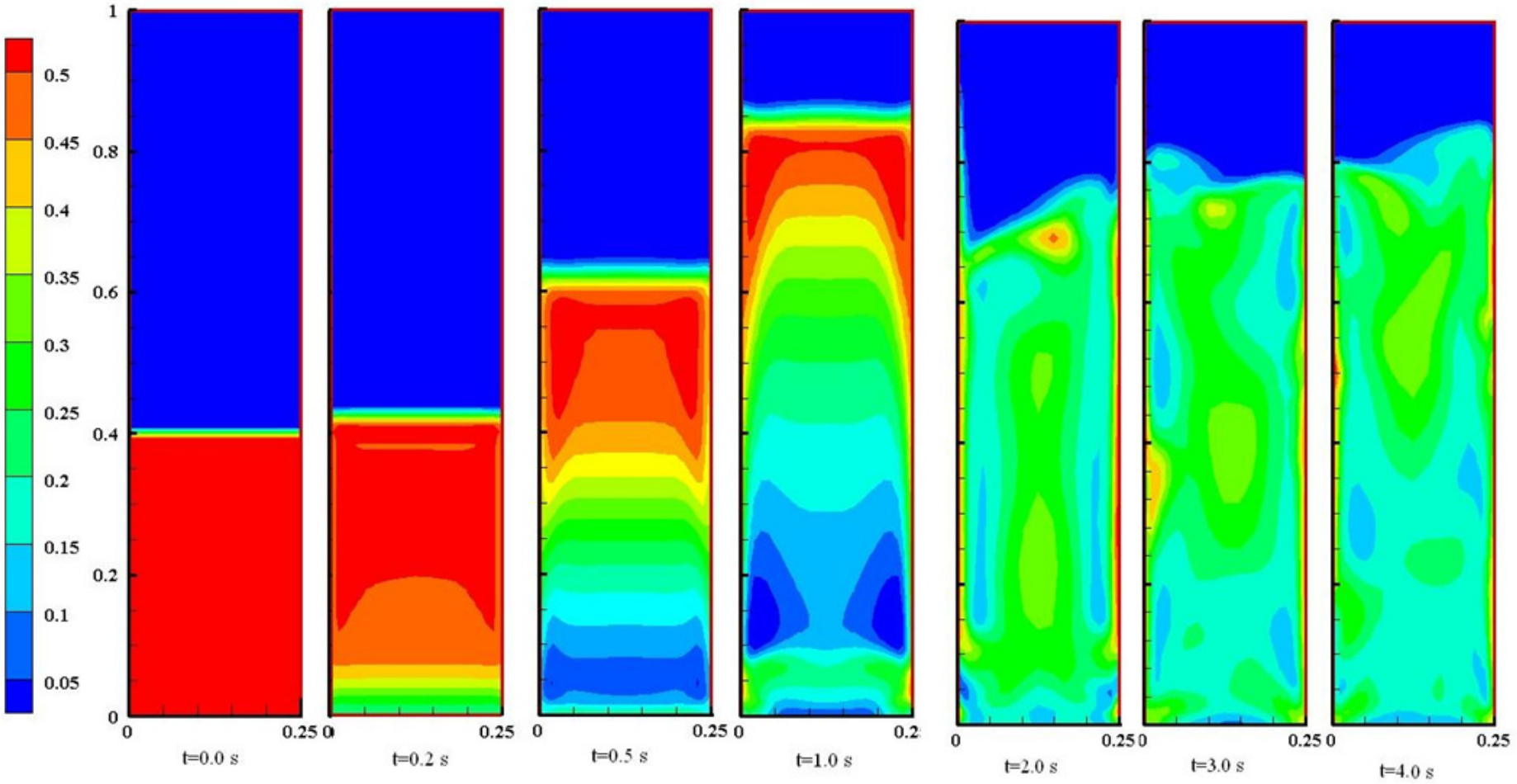


Solid Volume Fraction Contours

Clarkson University

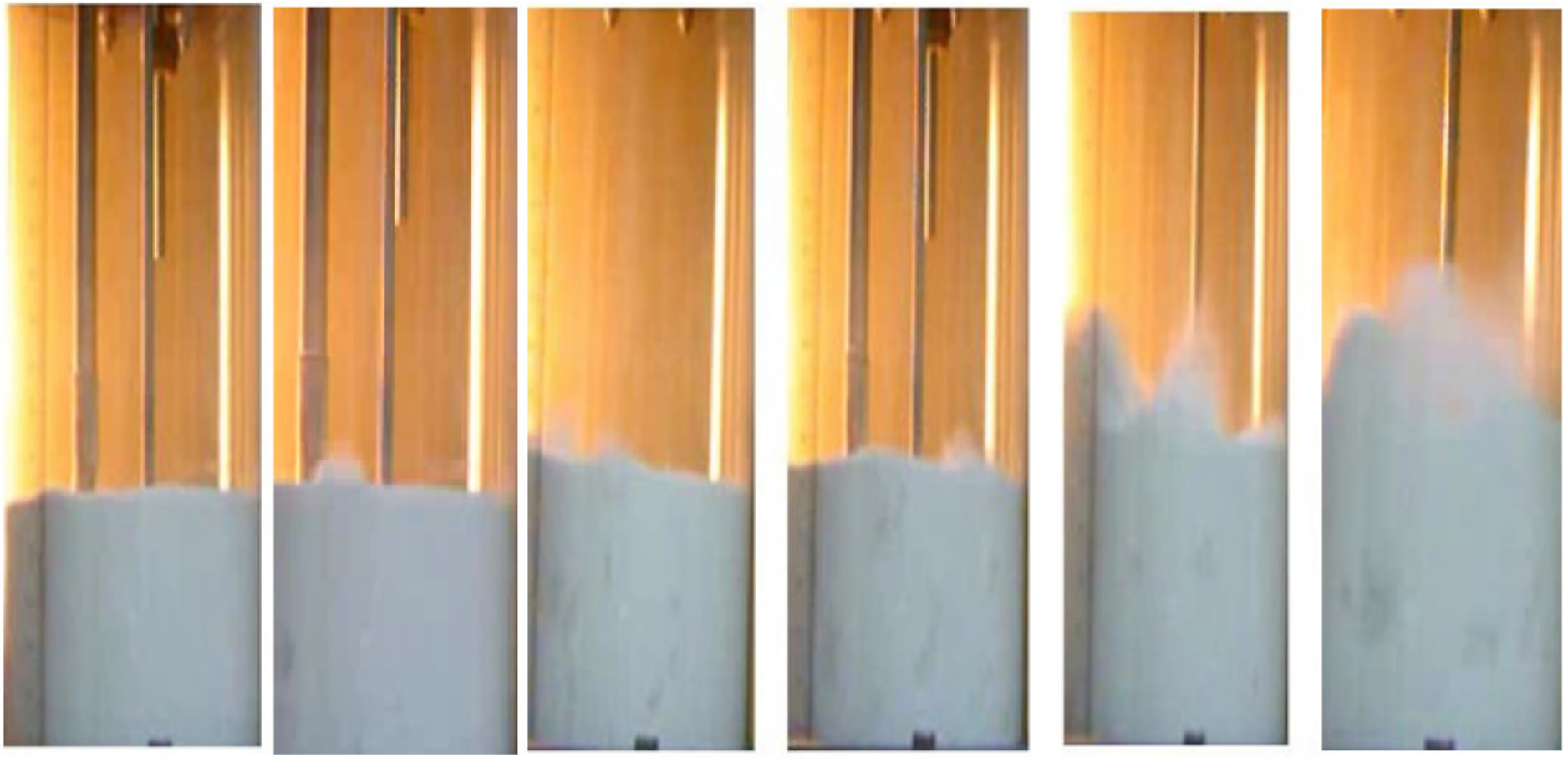
Superficial velocity V_g = 50 cm/s

Syamlal–O'Brien Drag



Bed Expansion vs V_g

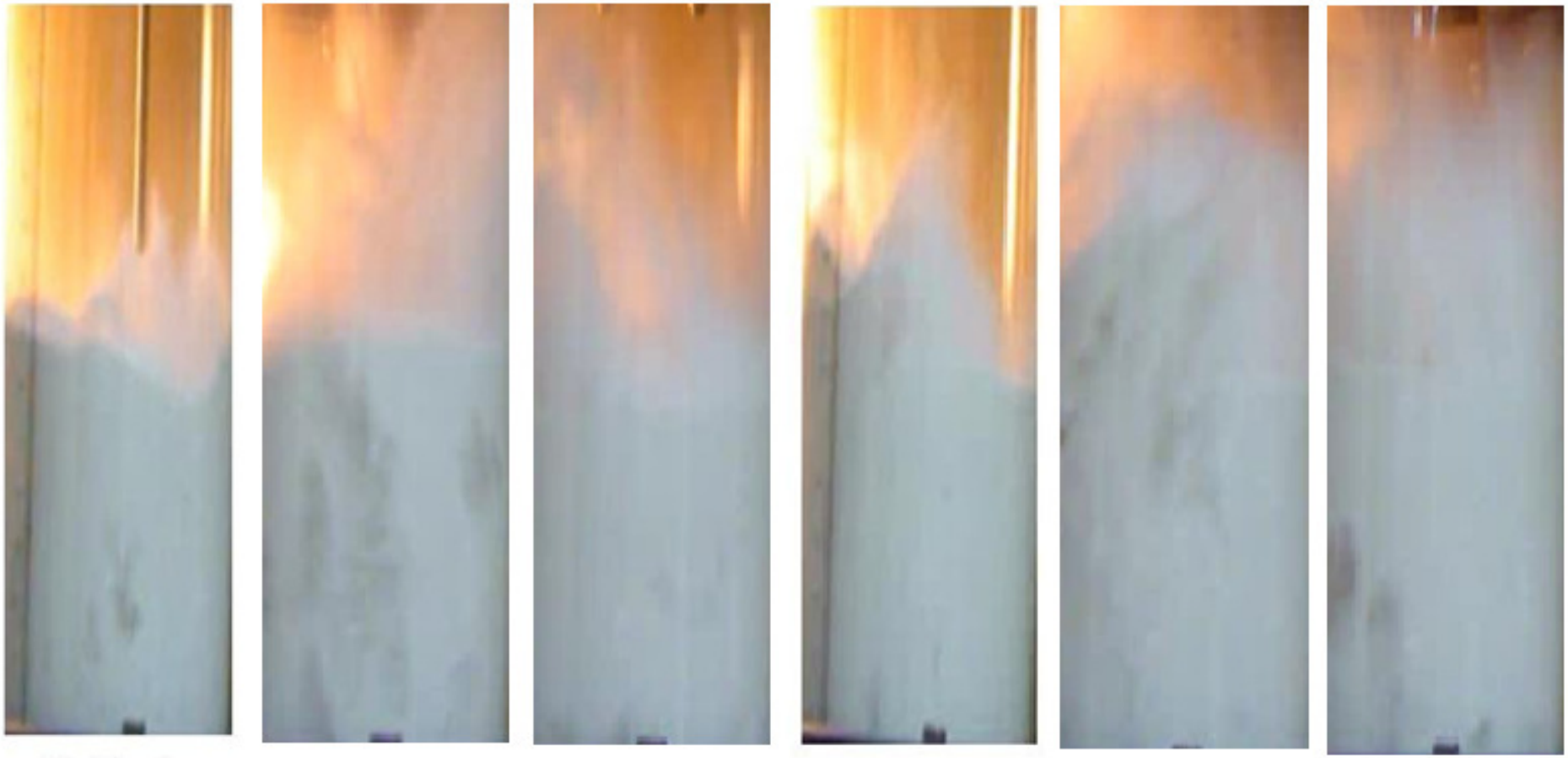
Clarkson University



Vg=0.0 cm/s Vg=7.0 cm/s Vg=10 cm/s Vg=20 cm/s Vg=35 cm/s Vg=40 cm/s

Bed Expansion vs V_g

Clarkson University

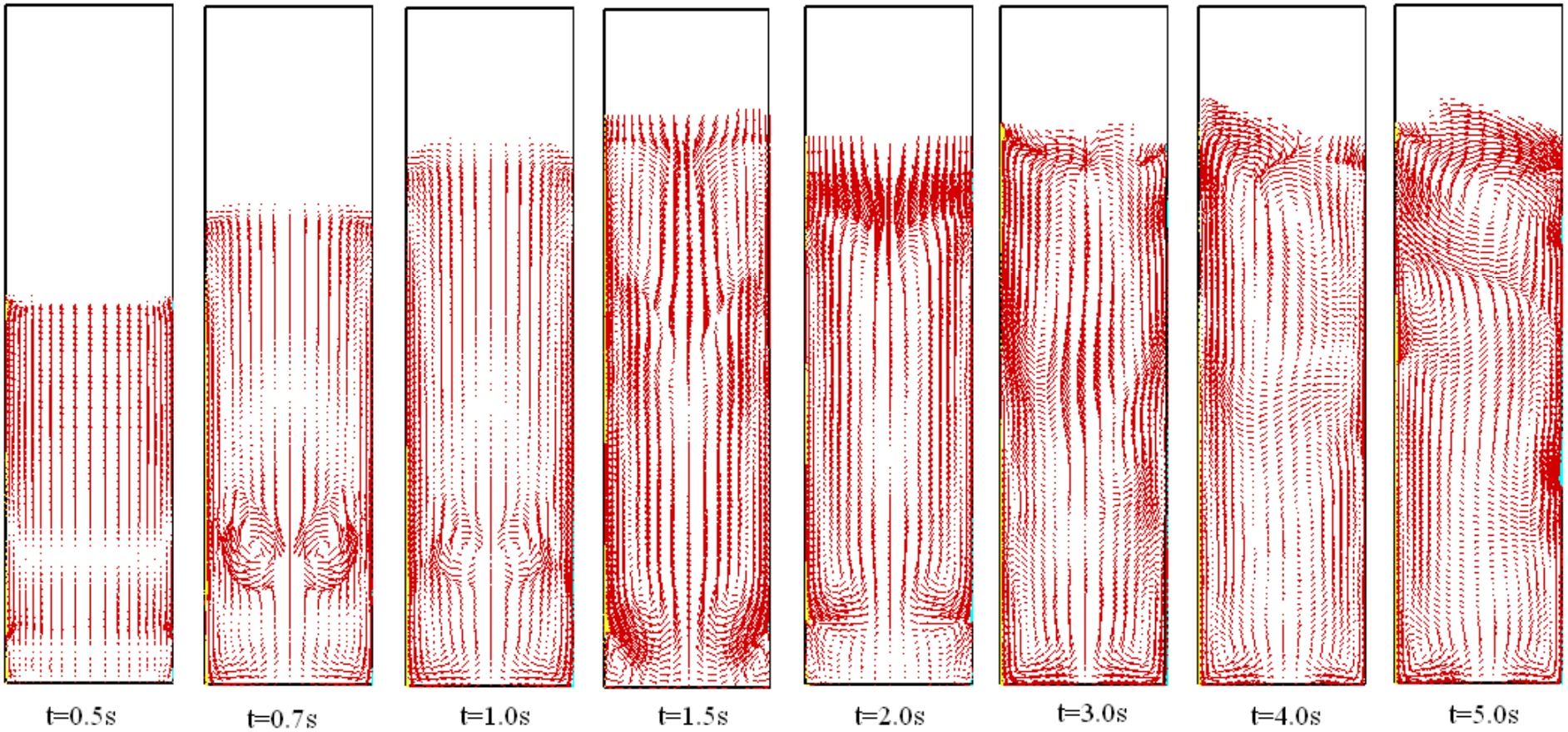


Vg=50 cm/s Vg=60 cm/s Vg=70 cm/s Vg=80 cm/s Vg=90 cm/s Vg=100 cm/s

Particle Velocity Field

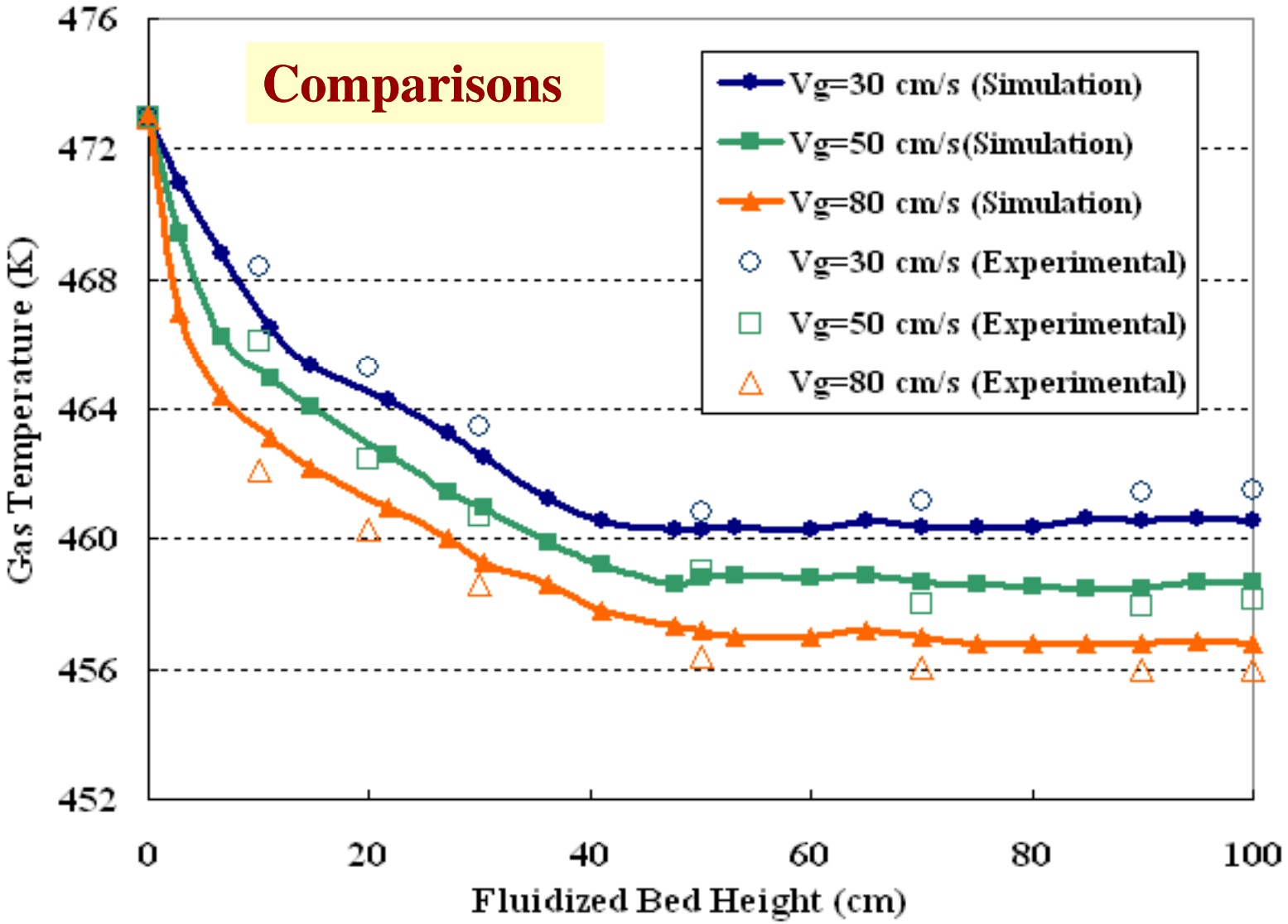
Clarkson University

Superficial velocity V_g = 50 cm/s



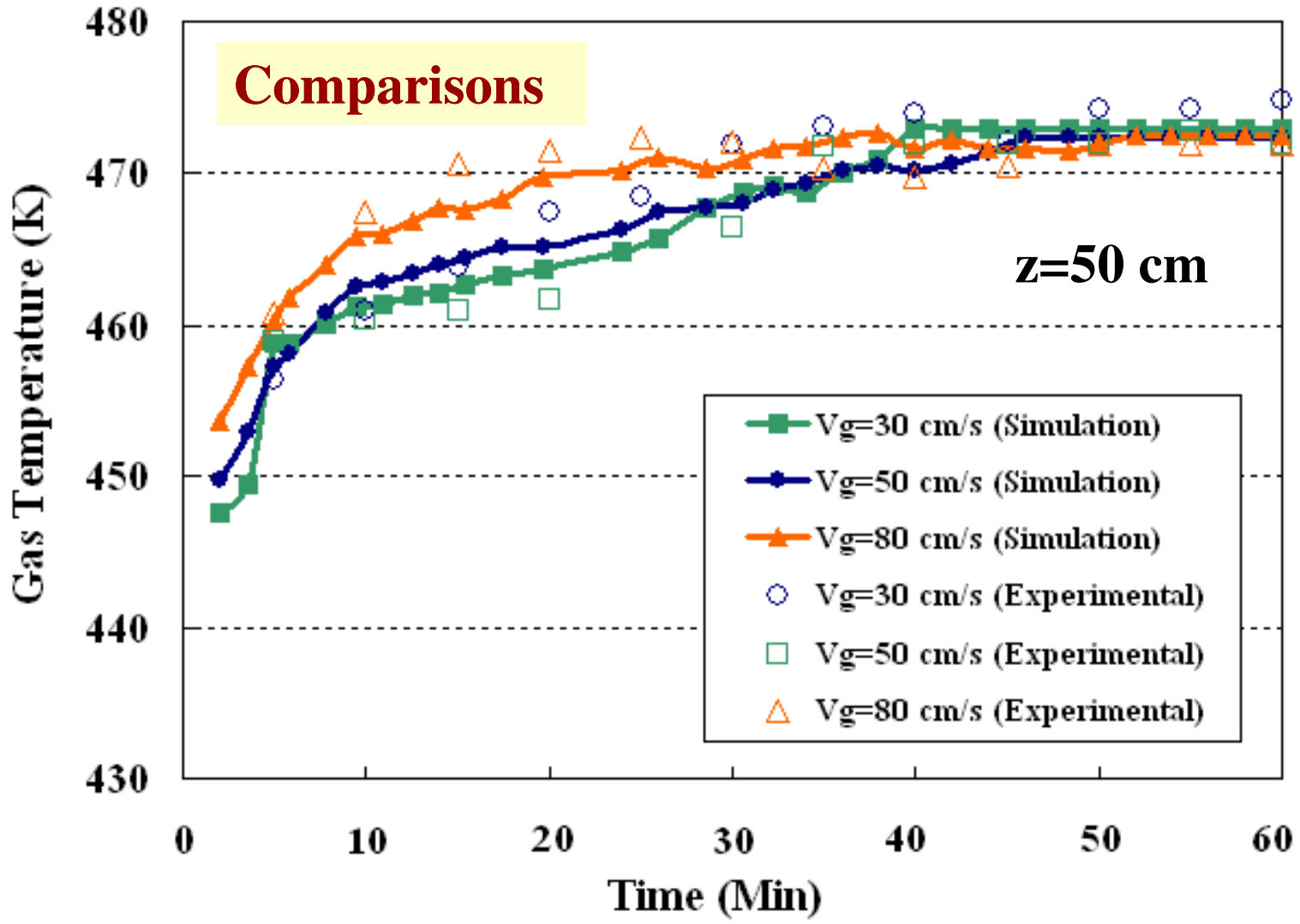
Gas Temperatures

Clarkson University



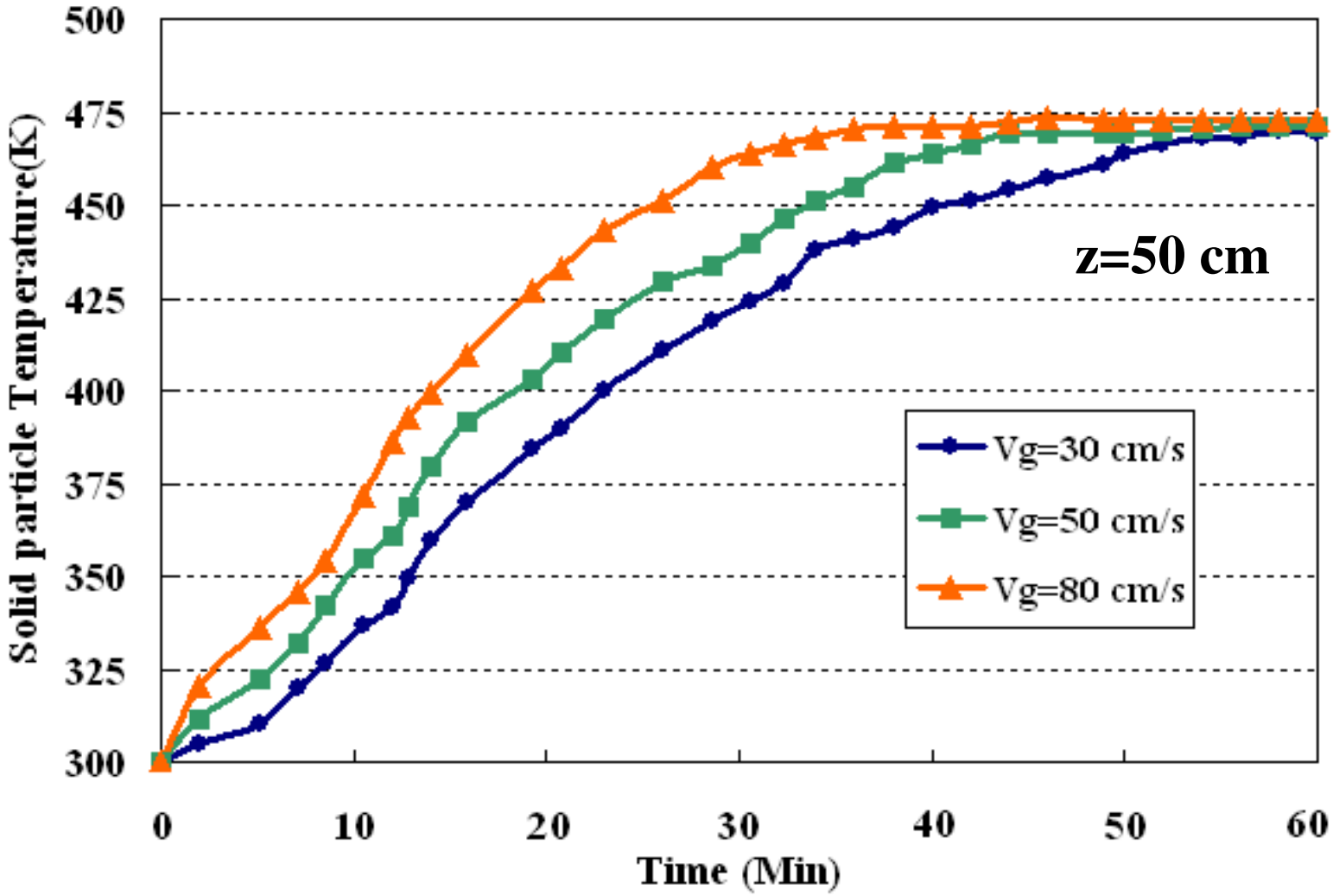
Gas Temperatures

Clarkson University



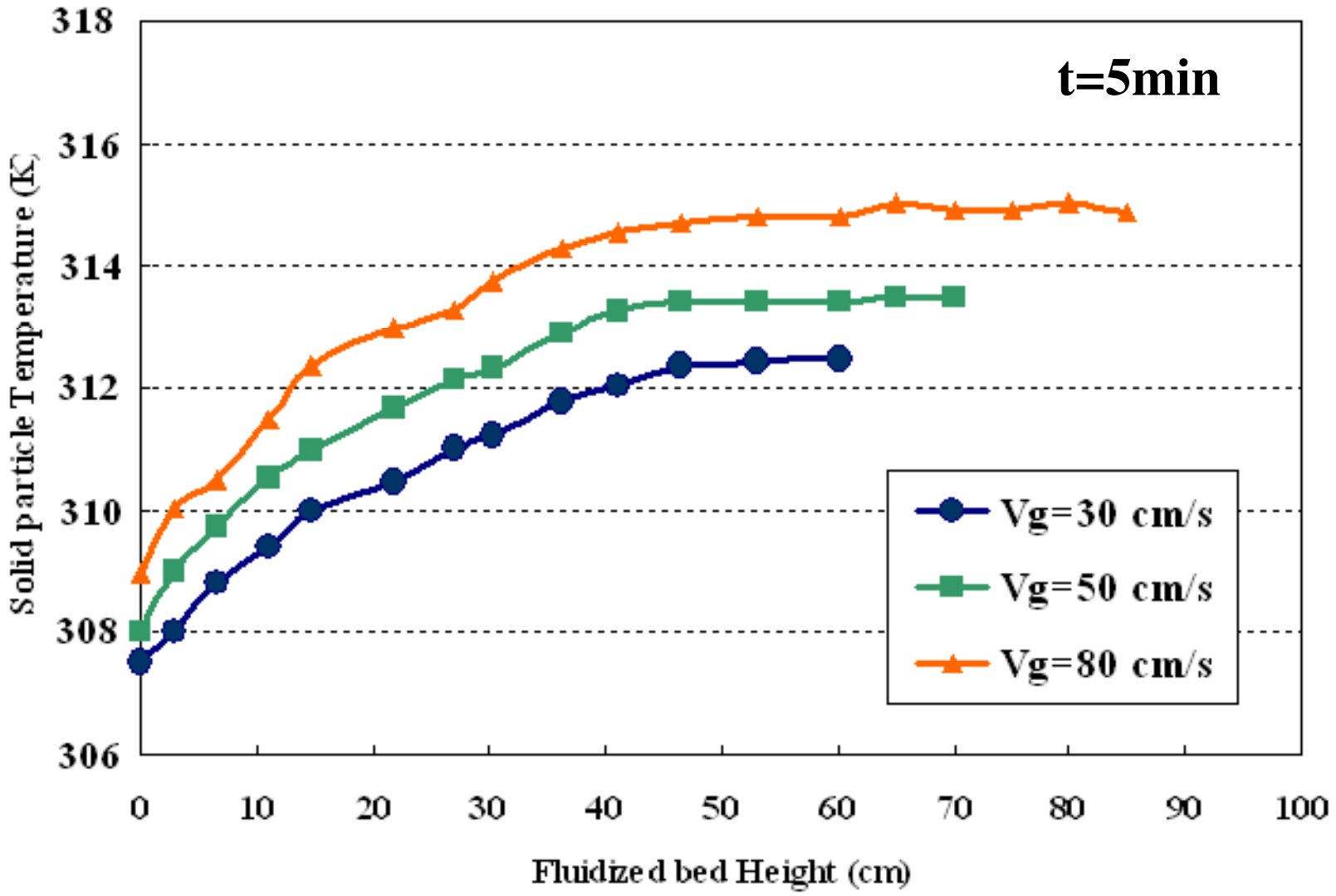
Particle Temperatures

Clarkson University



Particle Temperatures

Clarkson University



Conclusions

Clarkson University

- Unsteady flow and heat transfer in a gas–solid fluidized bed reactor was investigated.
- The Eulerian-Eulerian model with the standard k-ε turbulence model was used.
- The bed expansion ratio, pressure drop and voidage profiles were predicted.
- The model predictions agreed with the experimentally measured data.

Hamzehei, Rahimzadeh, Ahmadi (2010)

Powder Technology 200 (2010) 202–215

Contents lists available at ScienceDirect

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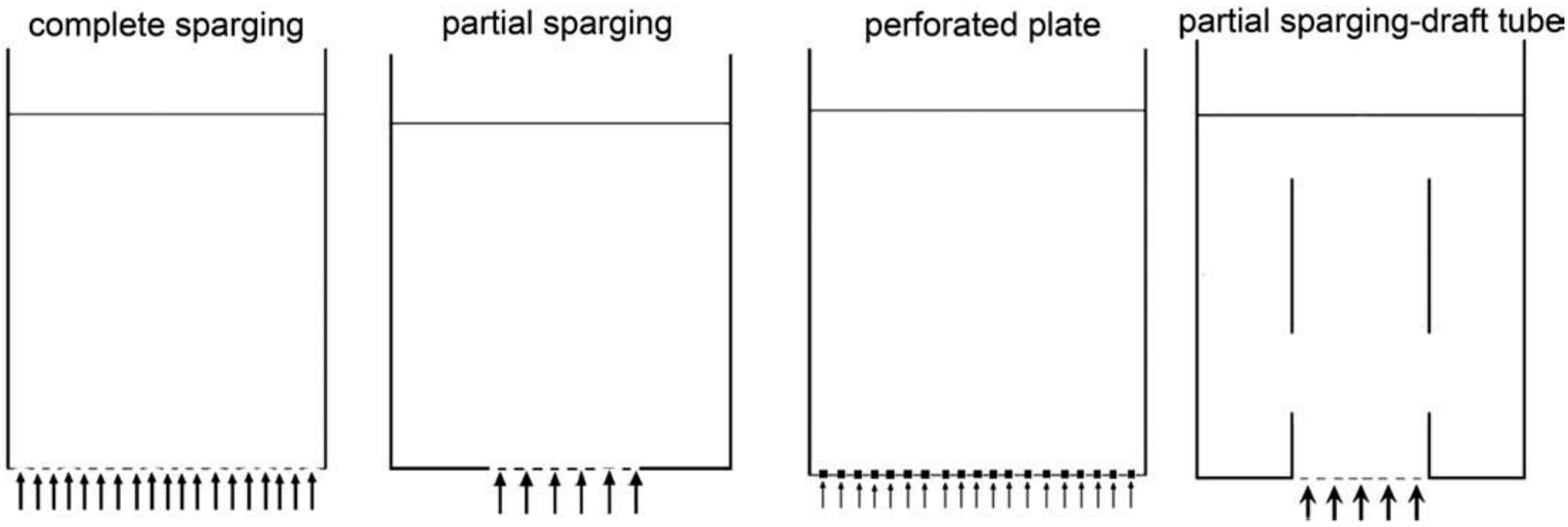
journal homepage: www.elsevier.com/locate/powtec

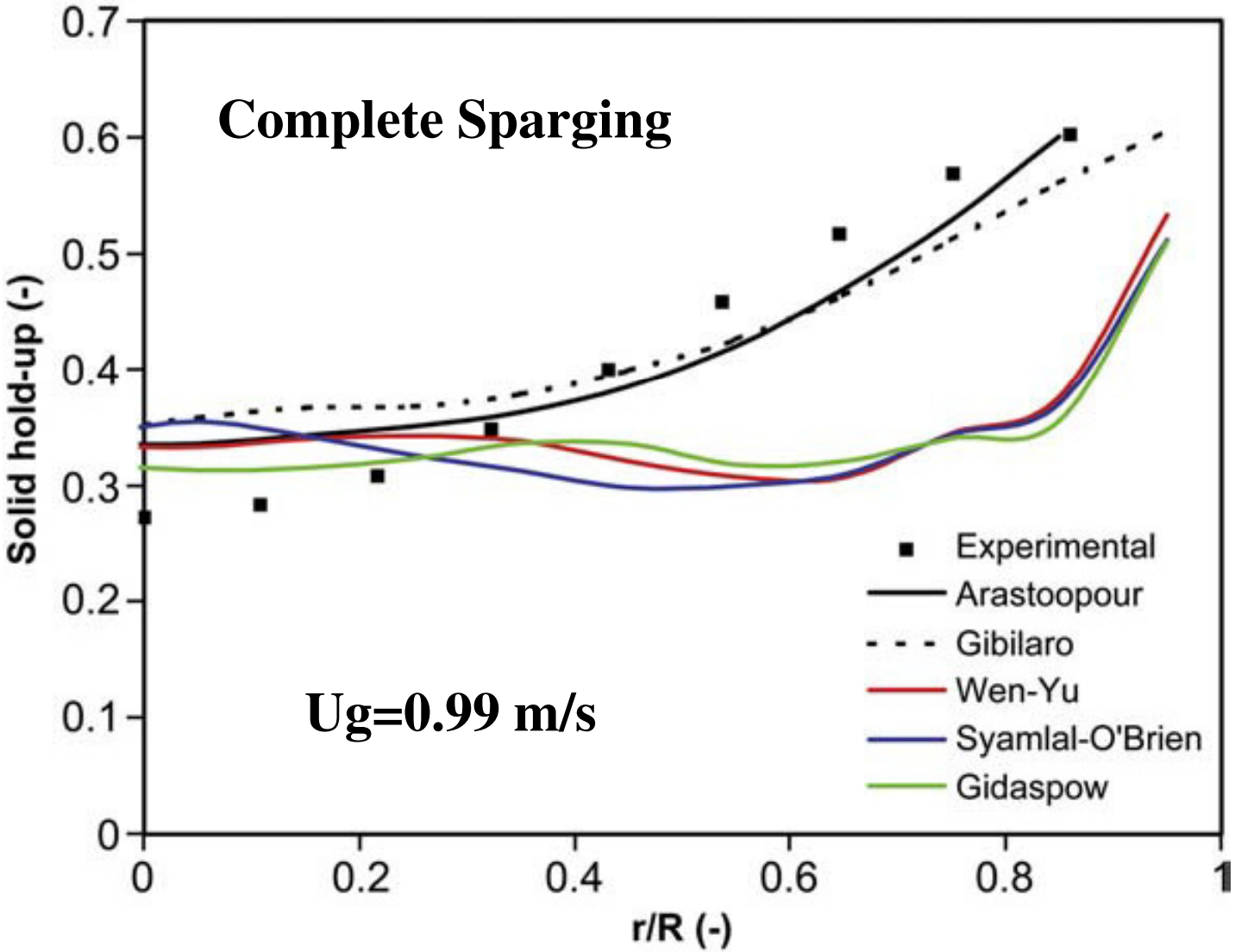
CFD studies of solids hold-up distribution and circulation patterns in gas–solid fluidized beds

Seyyed Hossein Hosseini [a,*], Goodarz Ahmadi [b], Rahbar Rahimi [c], Mortaza Zivdar [c], Mohsen Nasr Esfahany [d]



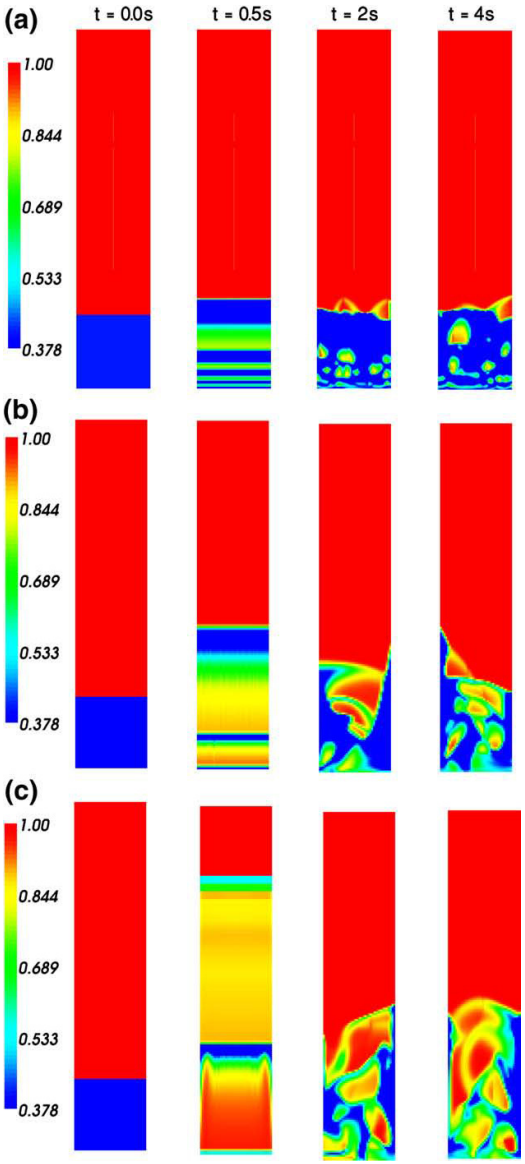
Solid Volume Fraction

Clarkson University



Gas Volume Fraction Contours

Clarkson University



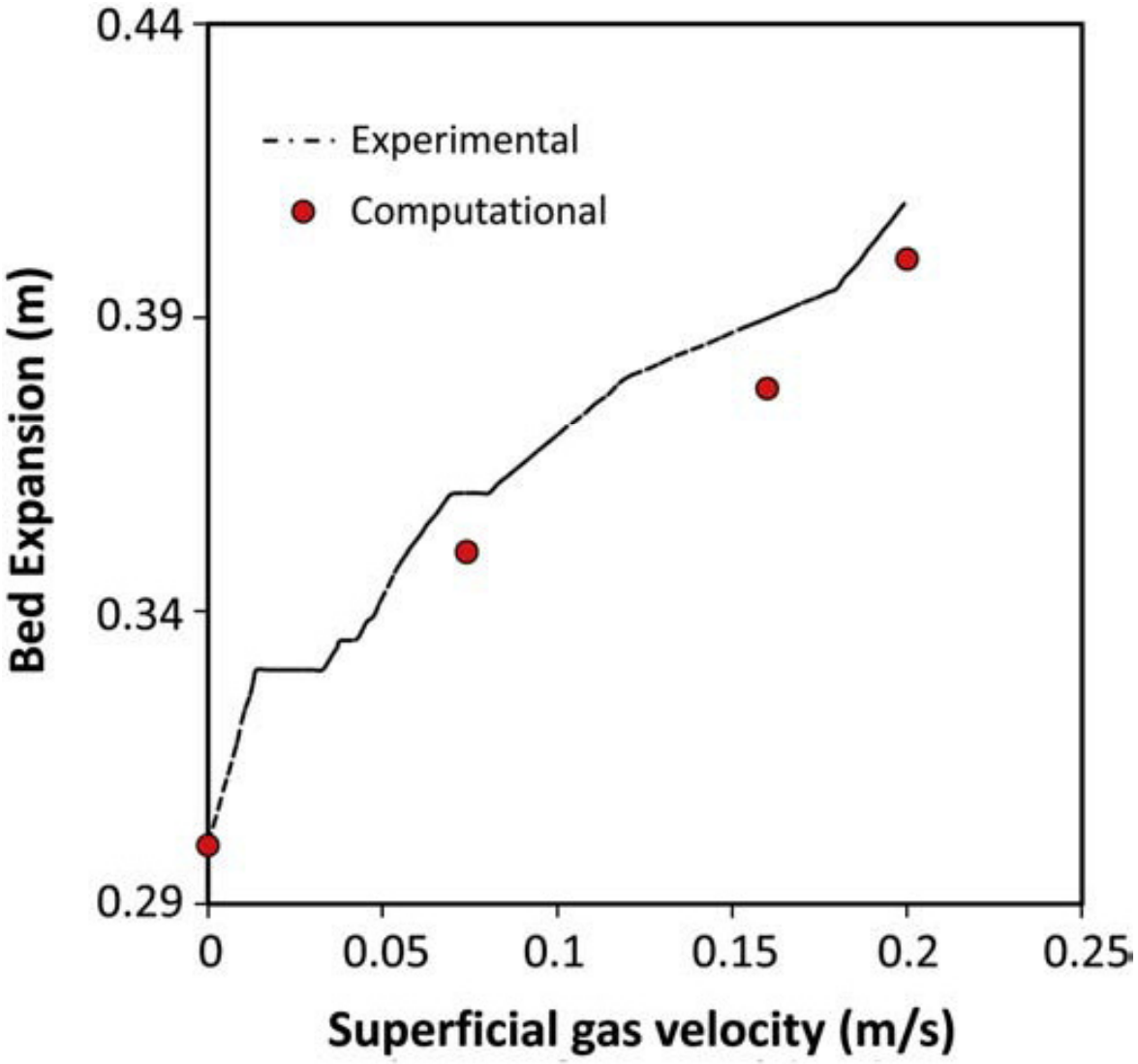
(a) Ug=0.35 m/s

(b) Ug=0.99 m/s

(c) Ug=2.18 m/s.

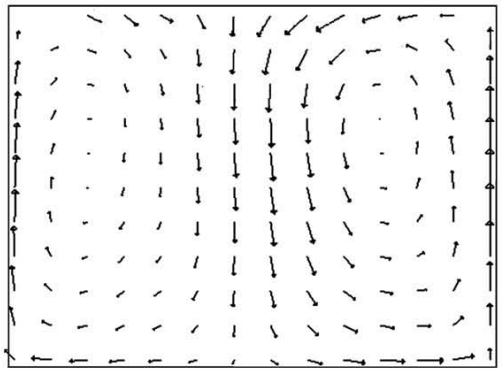
Bed Expansion

Clarkson University

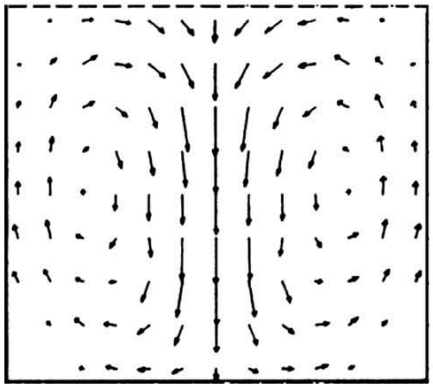


(a) Ug = 0.32 m/s

Computational

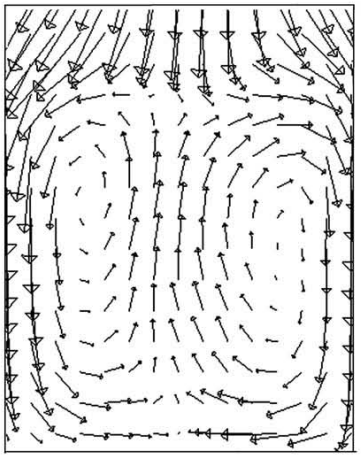


Lin et al. [37]

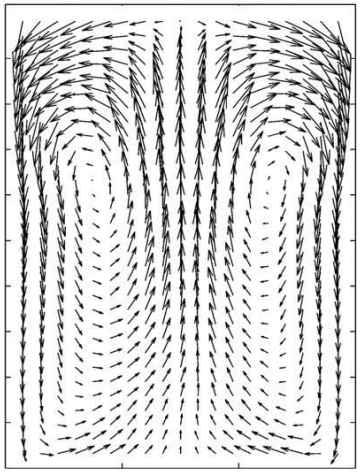


(b) Ug = 0.63 m/s

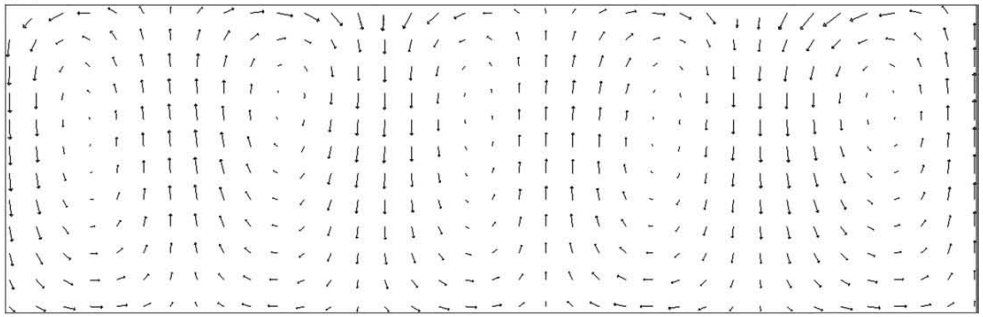
Computational



Laverman et al. [38]

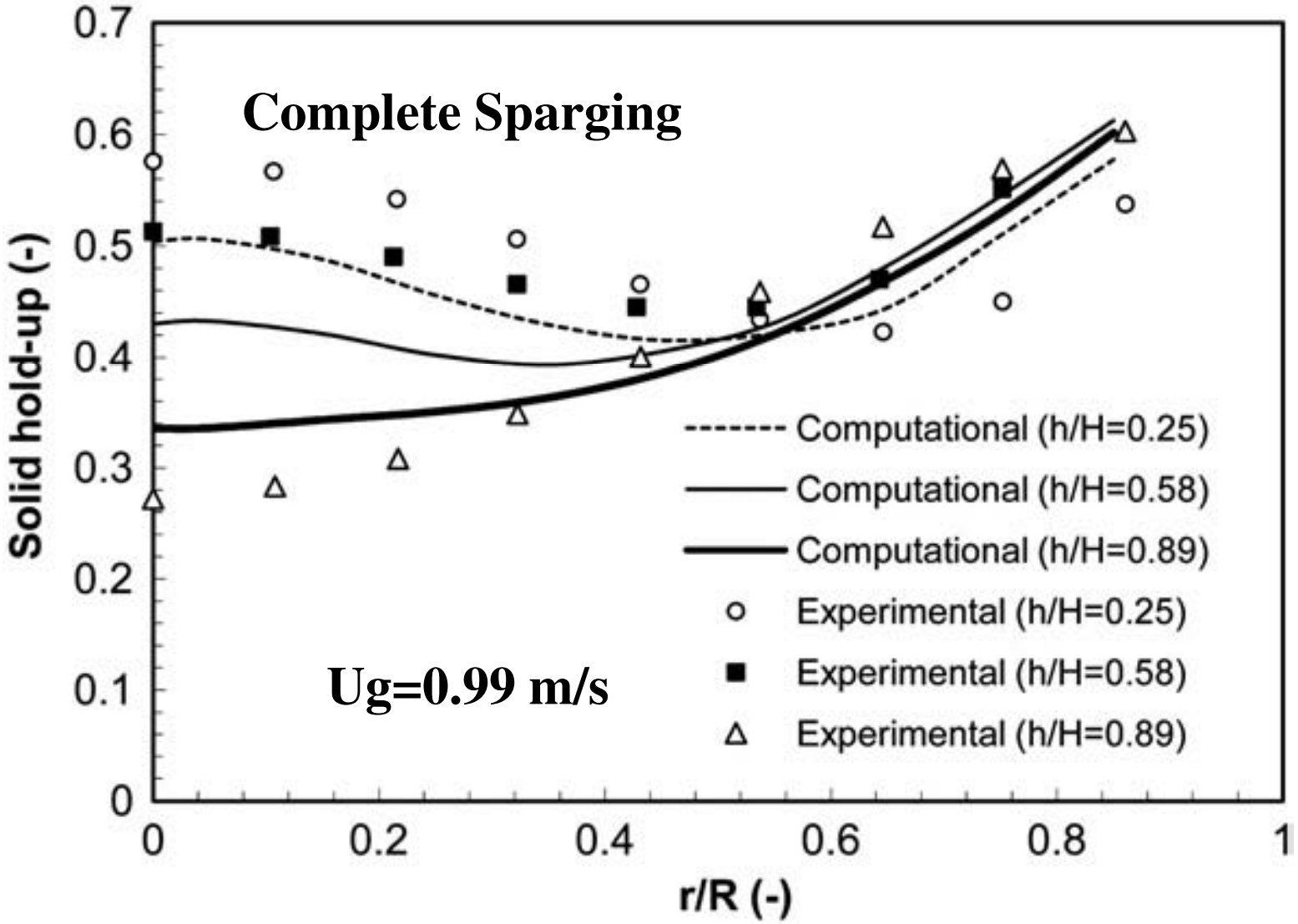


(c)



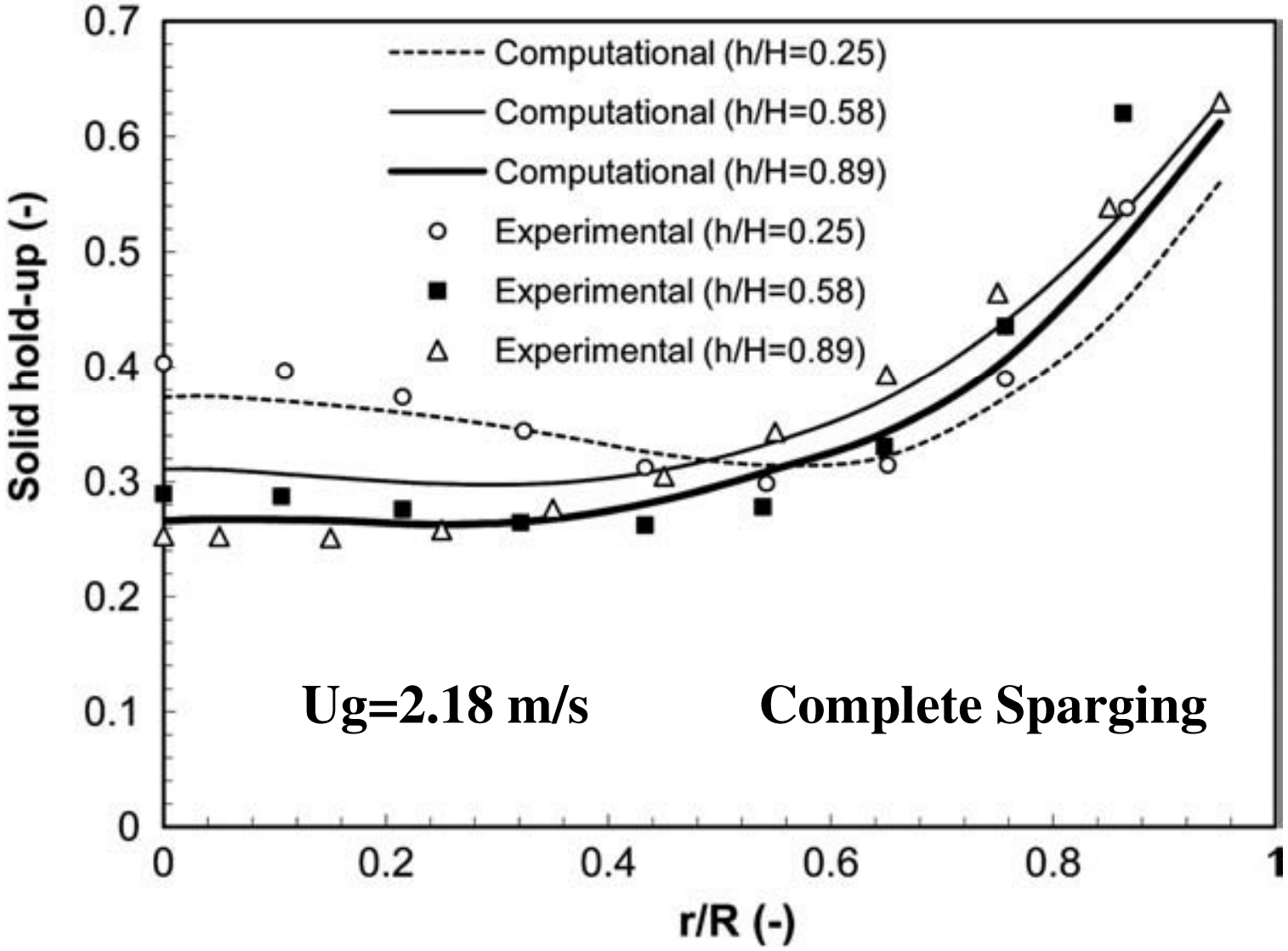
Solid Volume Fraction

Clarkson University



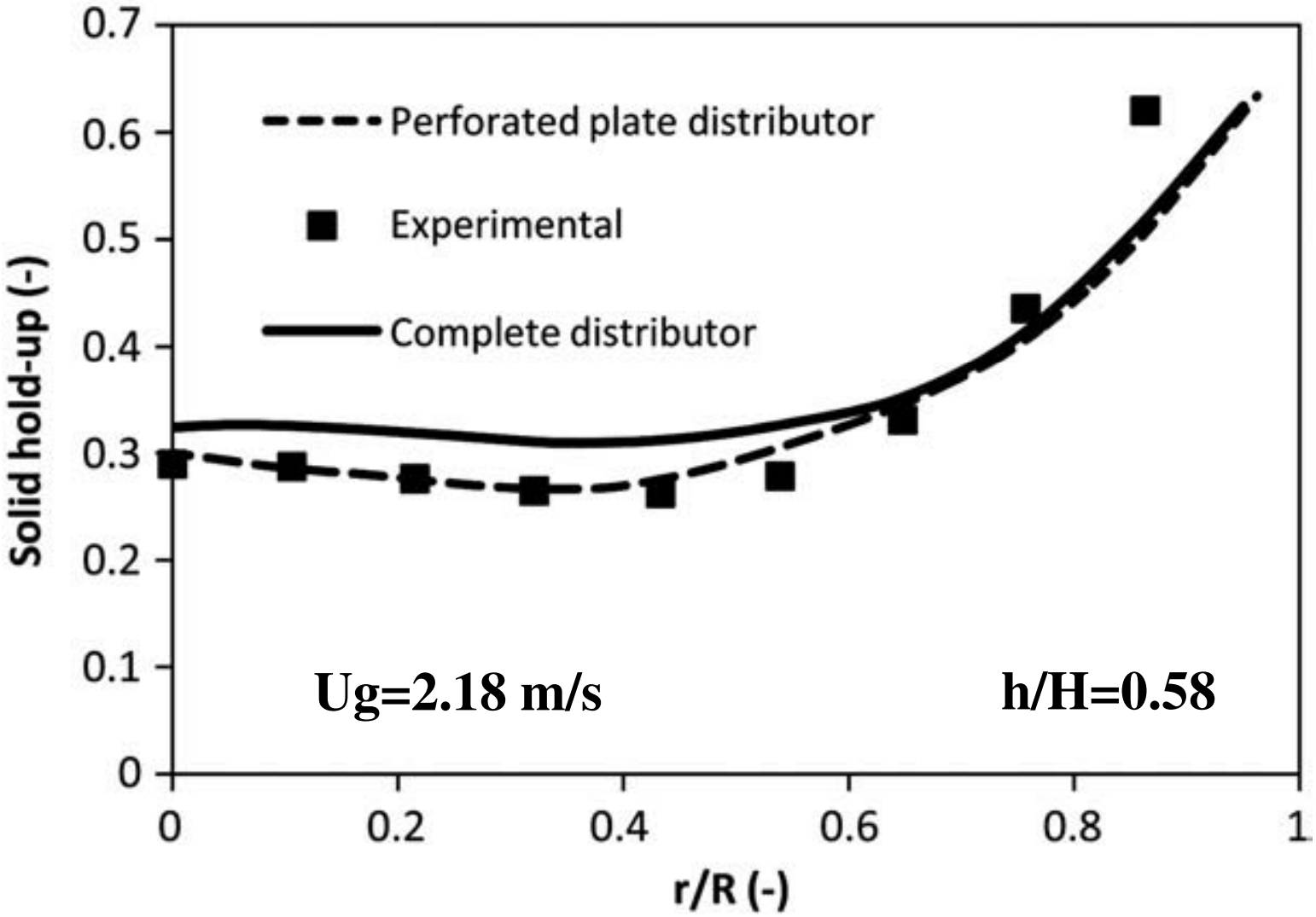
Solid Volume Fraction

Clarkson University



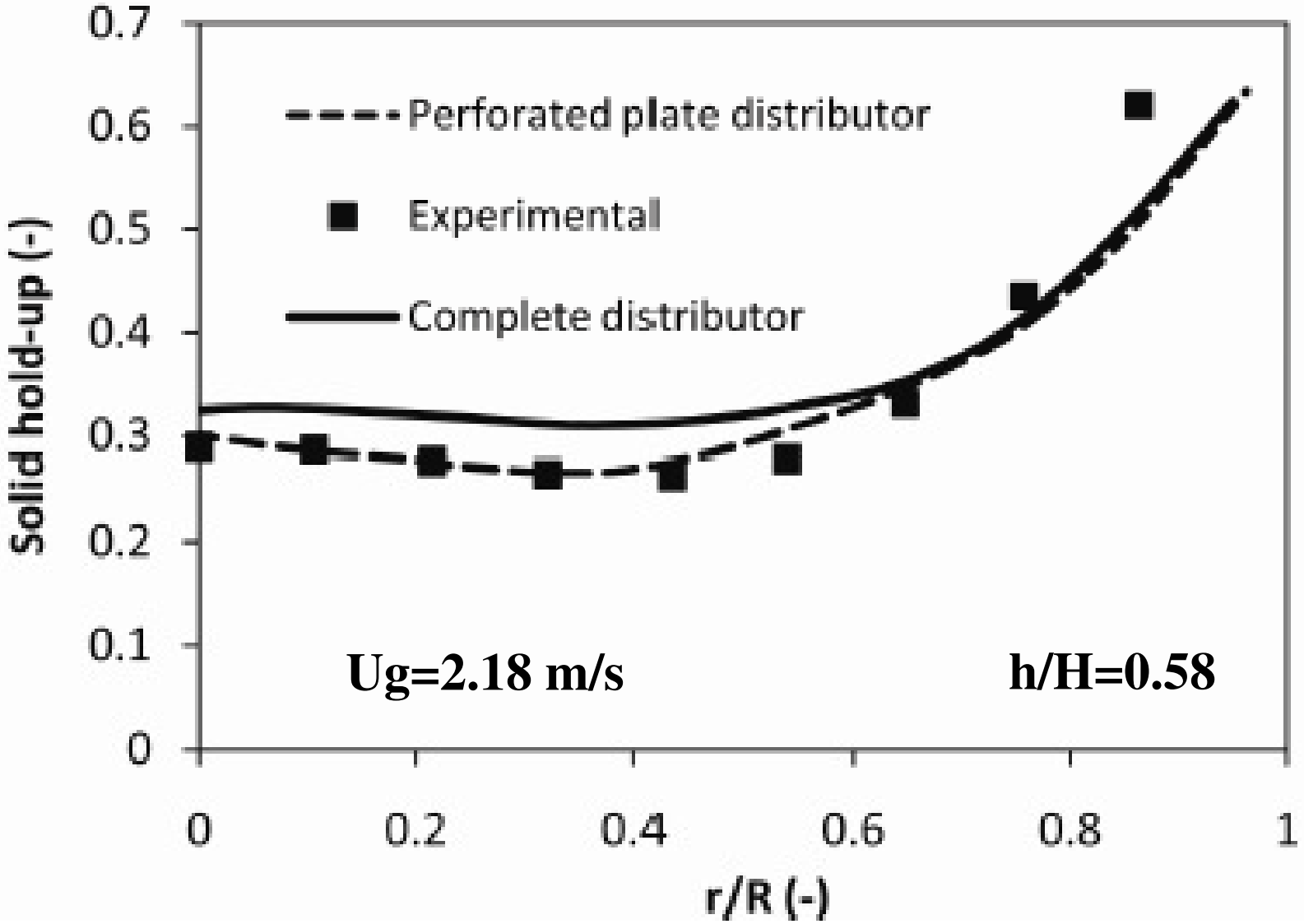
Solid Volume Fraction

Clarkson University



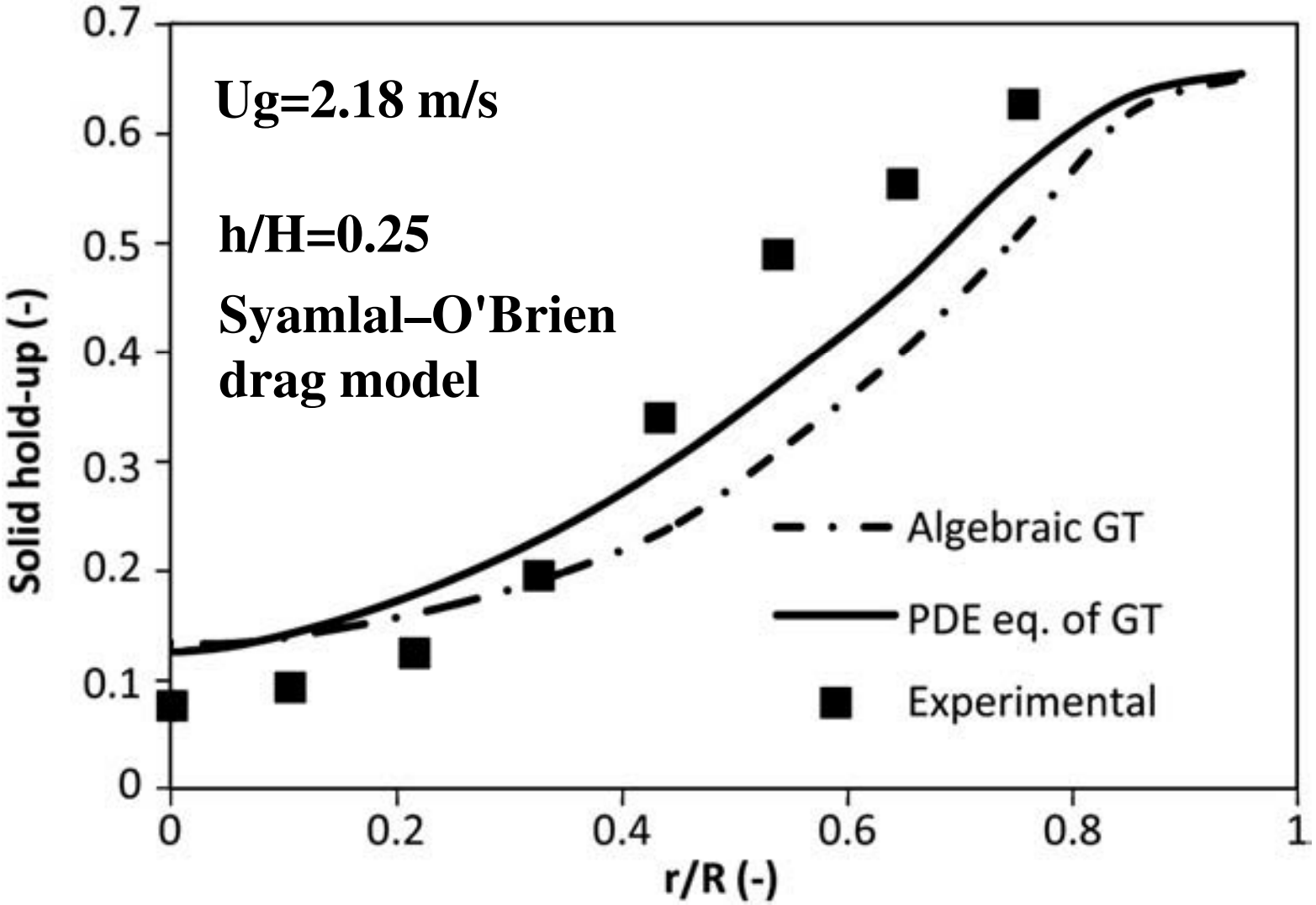
Solid Volume Fraction

Clarkson University



Solid Volume Fraction

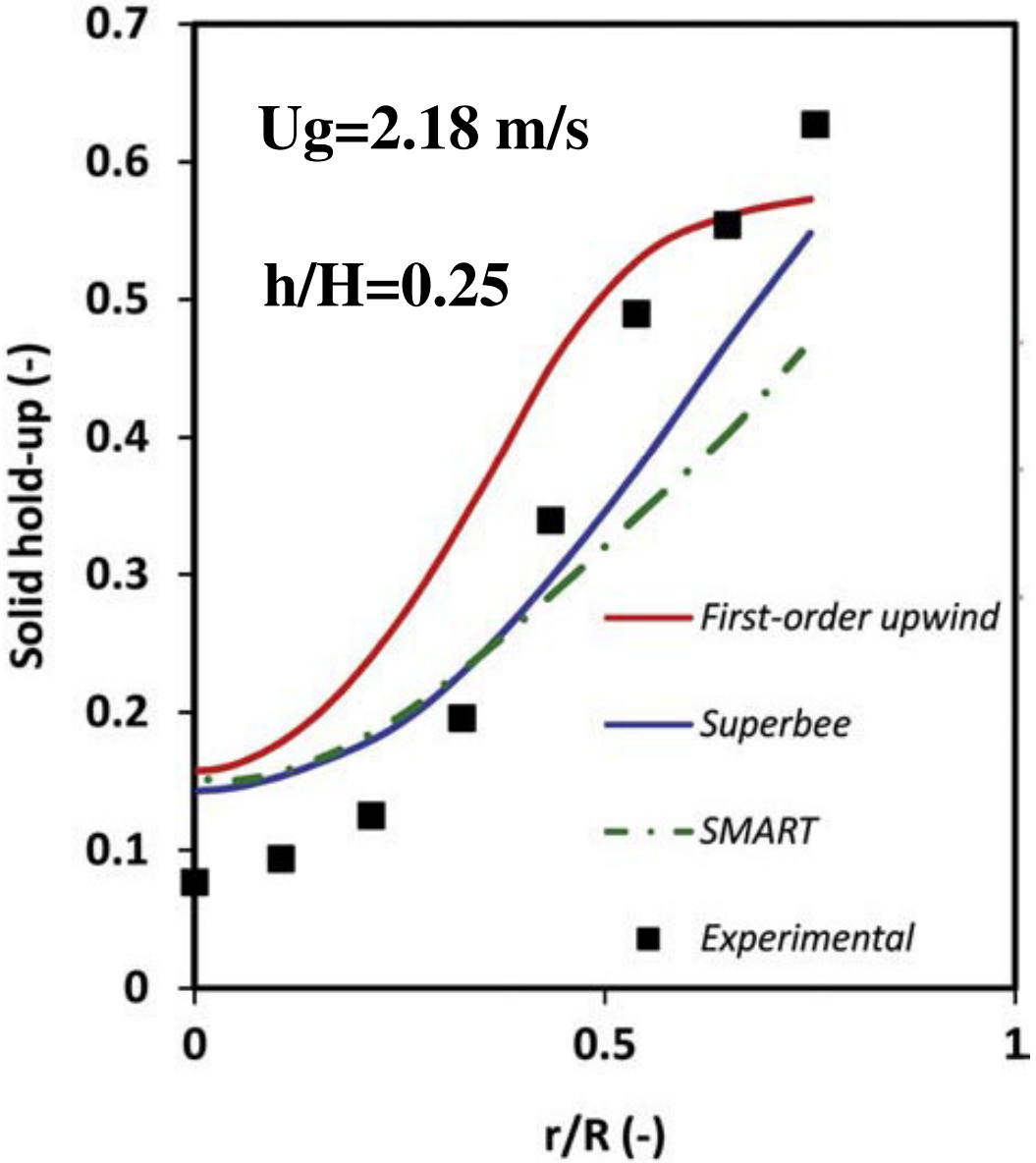
Clarkson University



Solid Volume Fraction

Clarkson University

Syamlal–O'Brien drag model



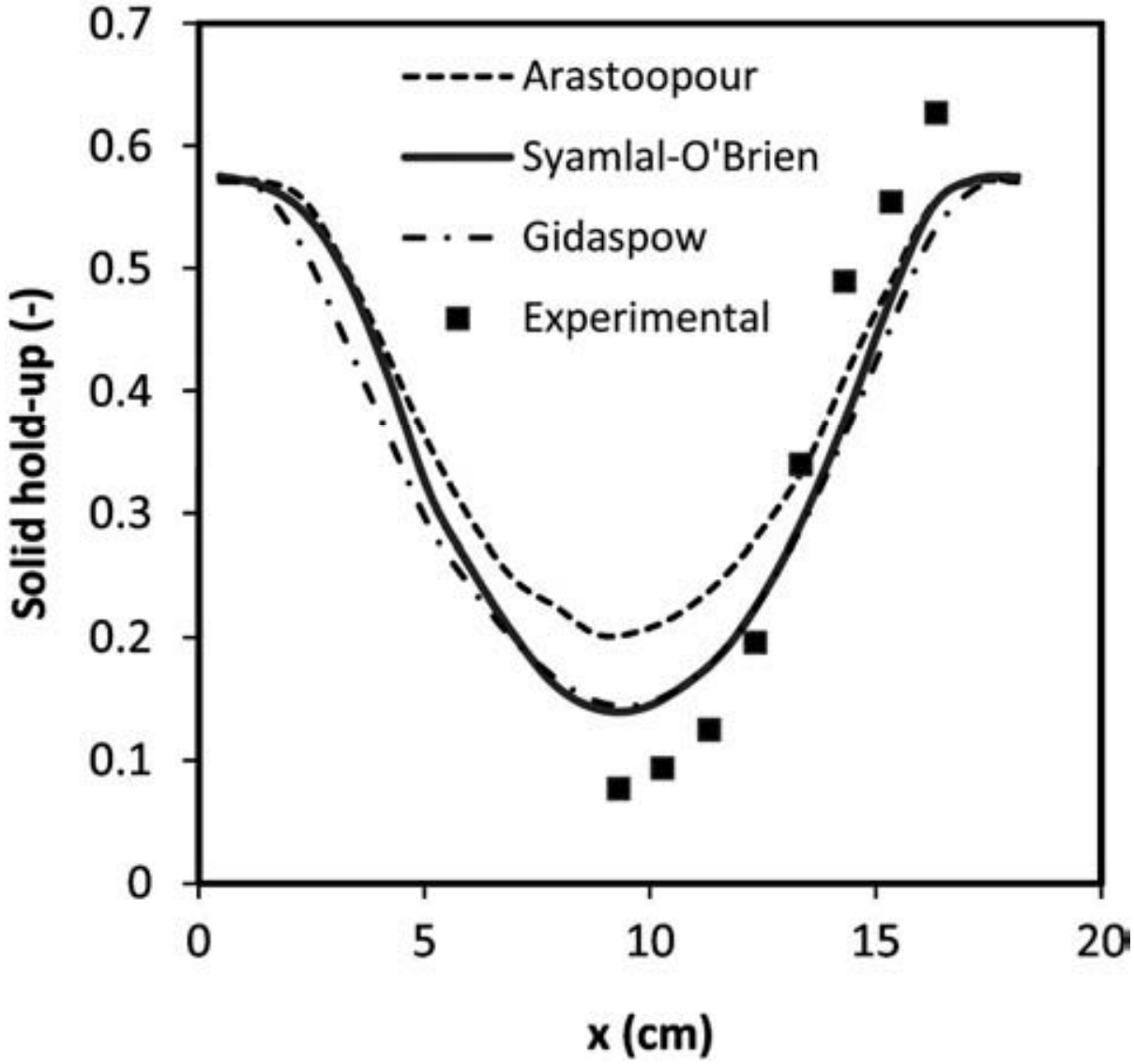
Solid Volume Fraction

Clarkson University

Ug=2.18 m/s

h/H=0.25

Partial Sparging



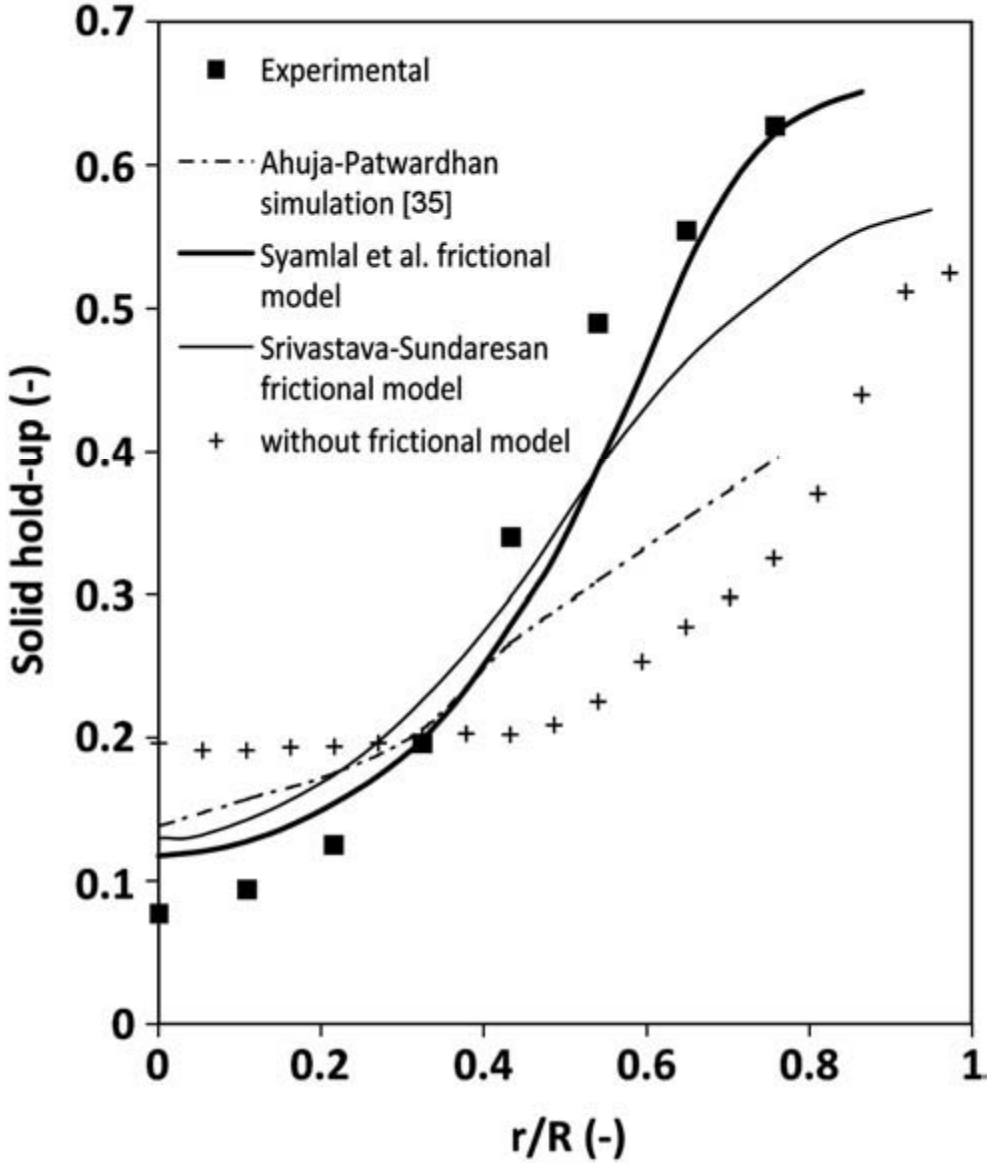
Solid Volume Fraction

Clarkson University

Ug=2.18 m/s

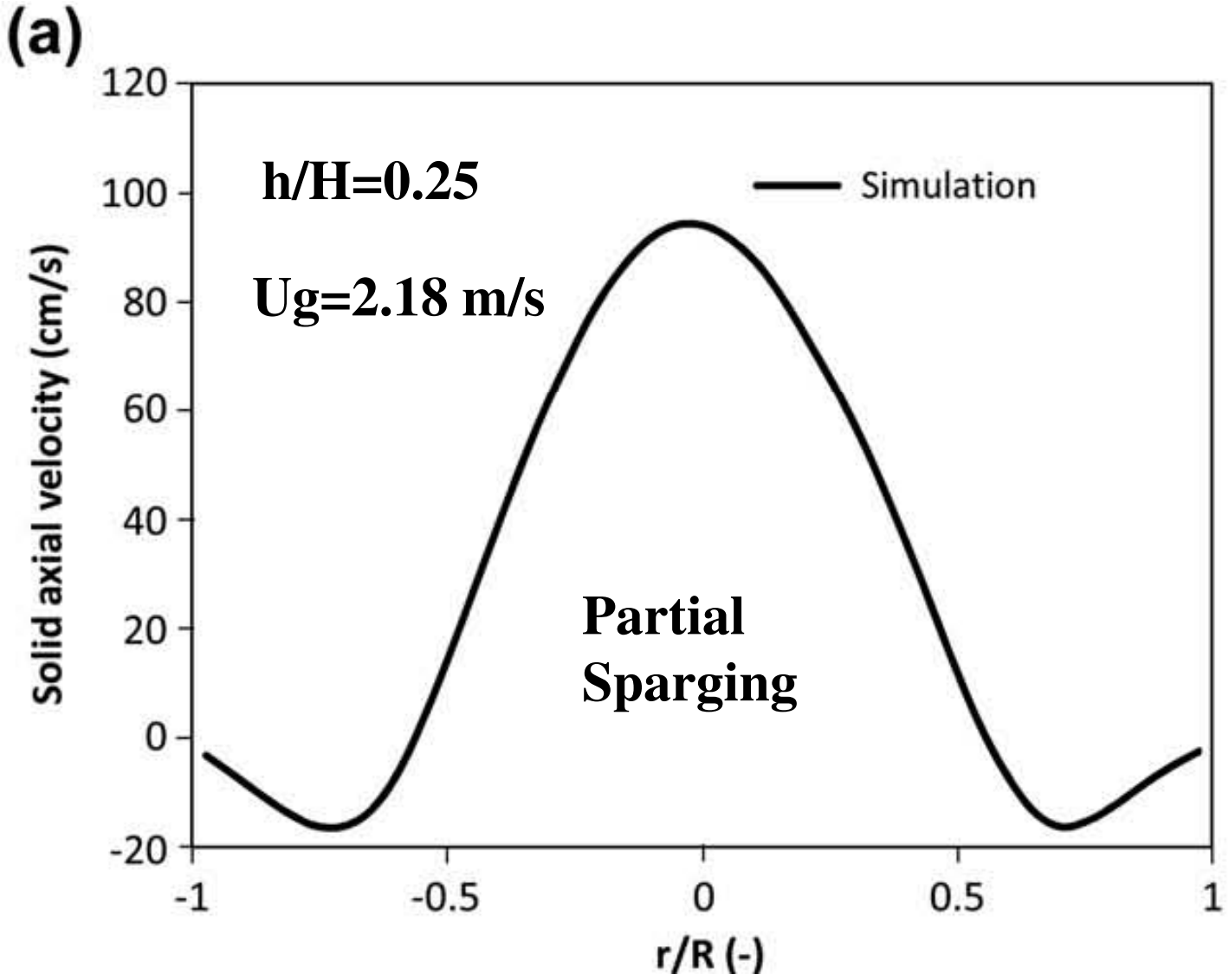
h/H=0.25

Partial Sparging



Solid Axial Velocity

Clarkson University

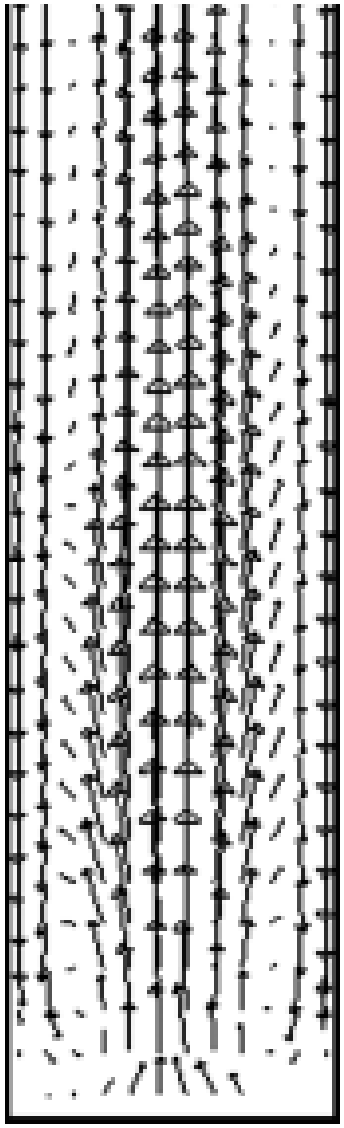


Solid Velocity Vector

Clarkson University

Ug=2.18 m/s

Partial Sparging

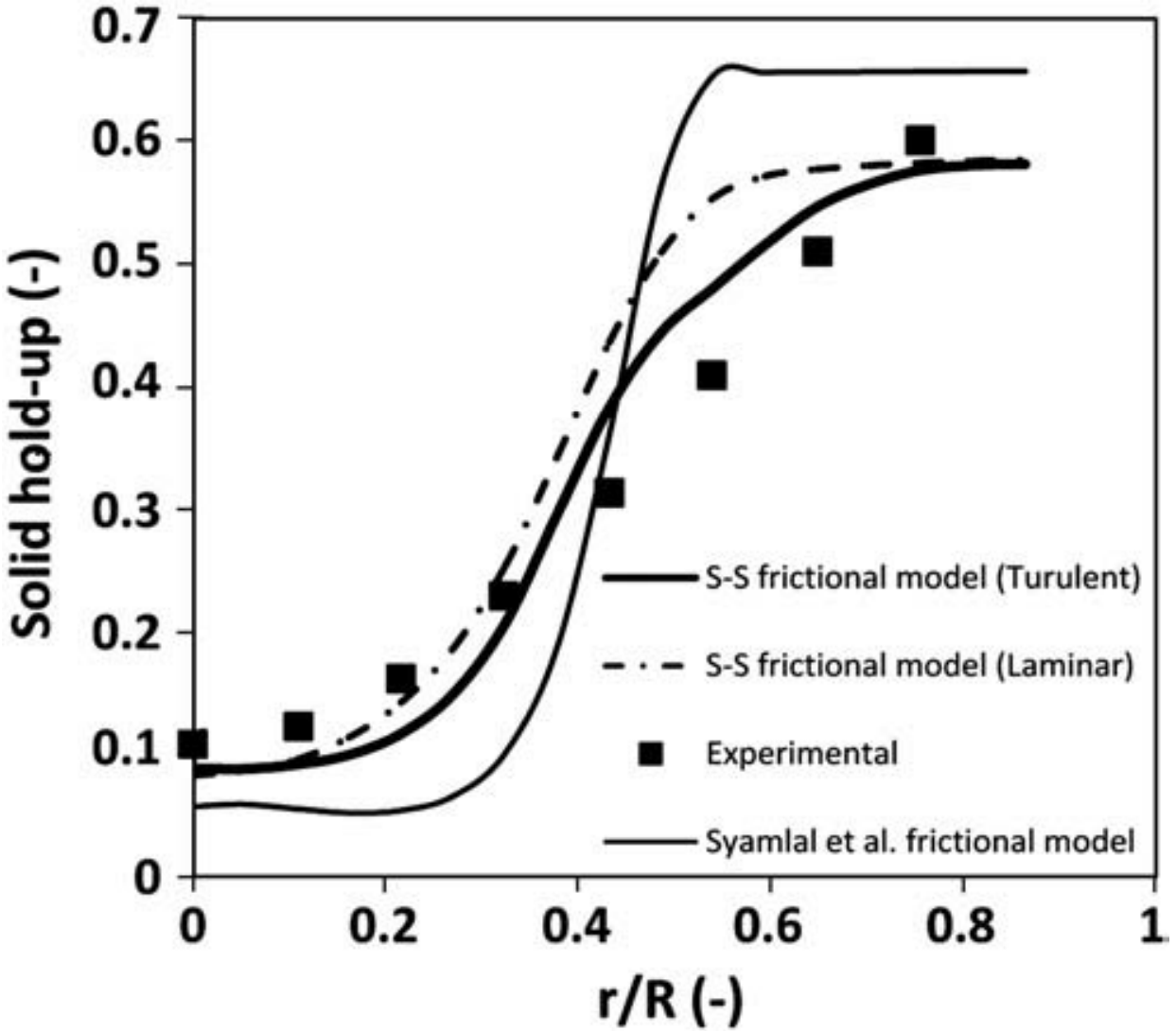


Solid Volume Fraction

Clarkson University

Ug=2.18 m/s

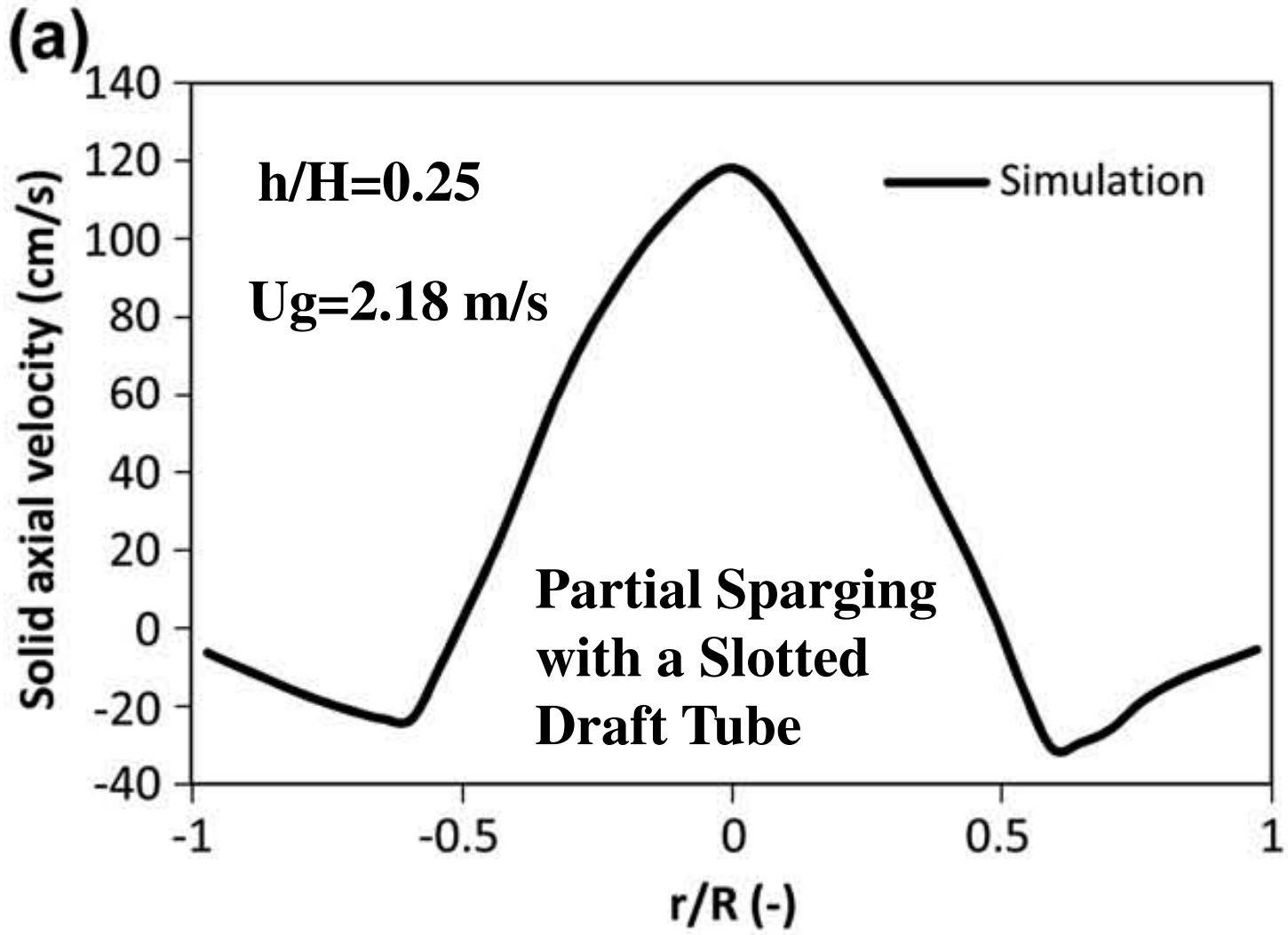
h/H=0.25



Partial Sparging with a Slotted Draft Tube

Solid Axial Velocity

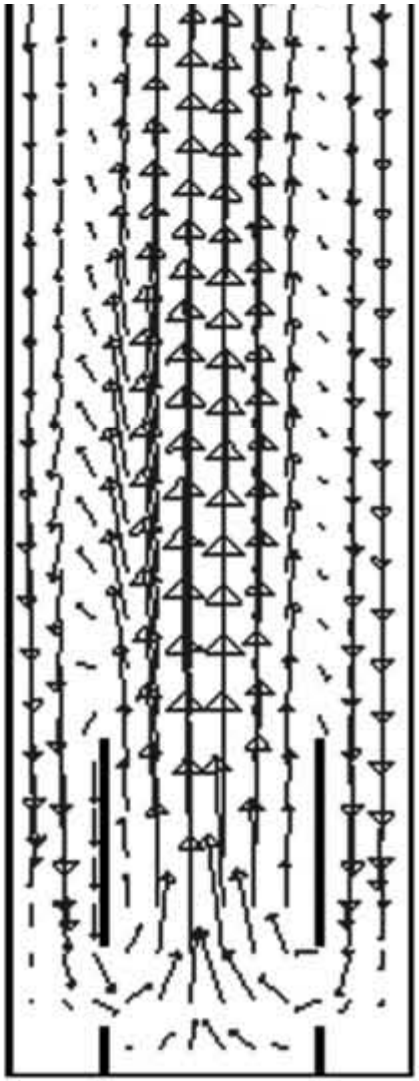
Clarkson University



Solid Velocity Vector

Clarkson University

Ug=2.18 m/s



Partial Sparging with a Slotted Draft Tube

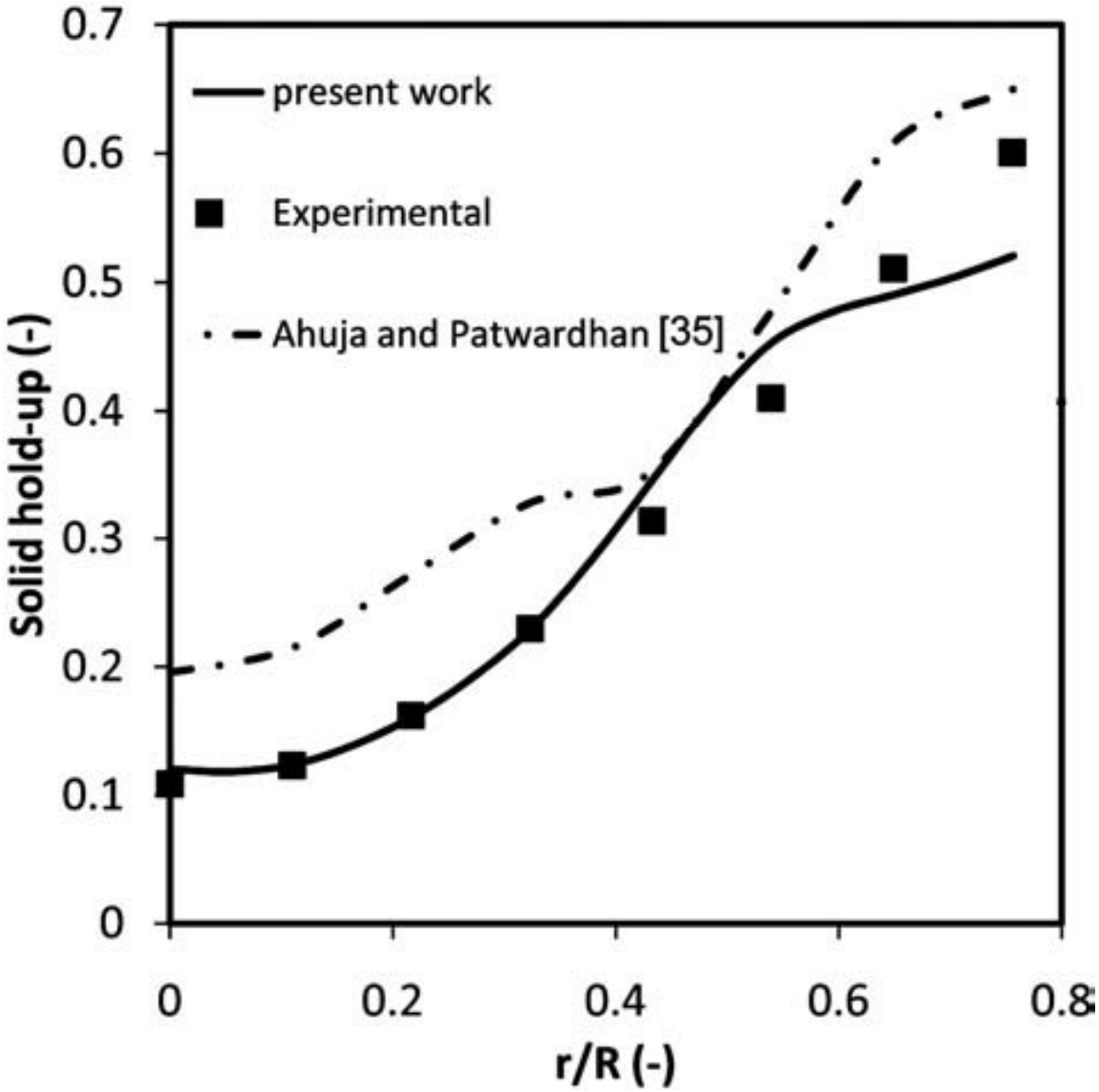
Solid Volume Fraction

Clarkson University

Ug=2.18 m/s

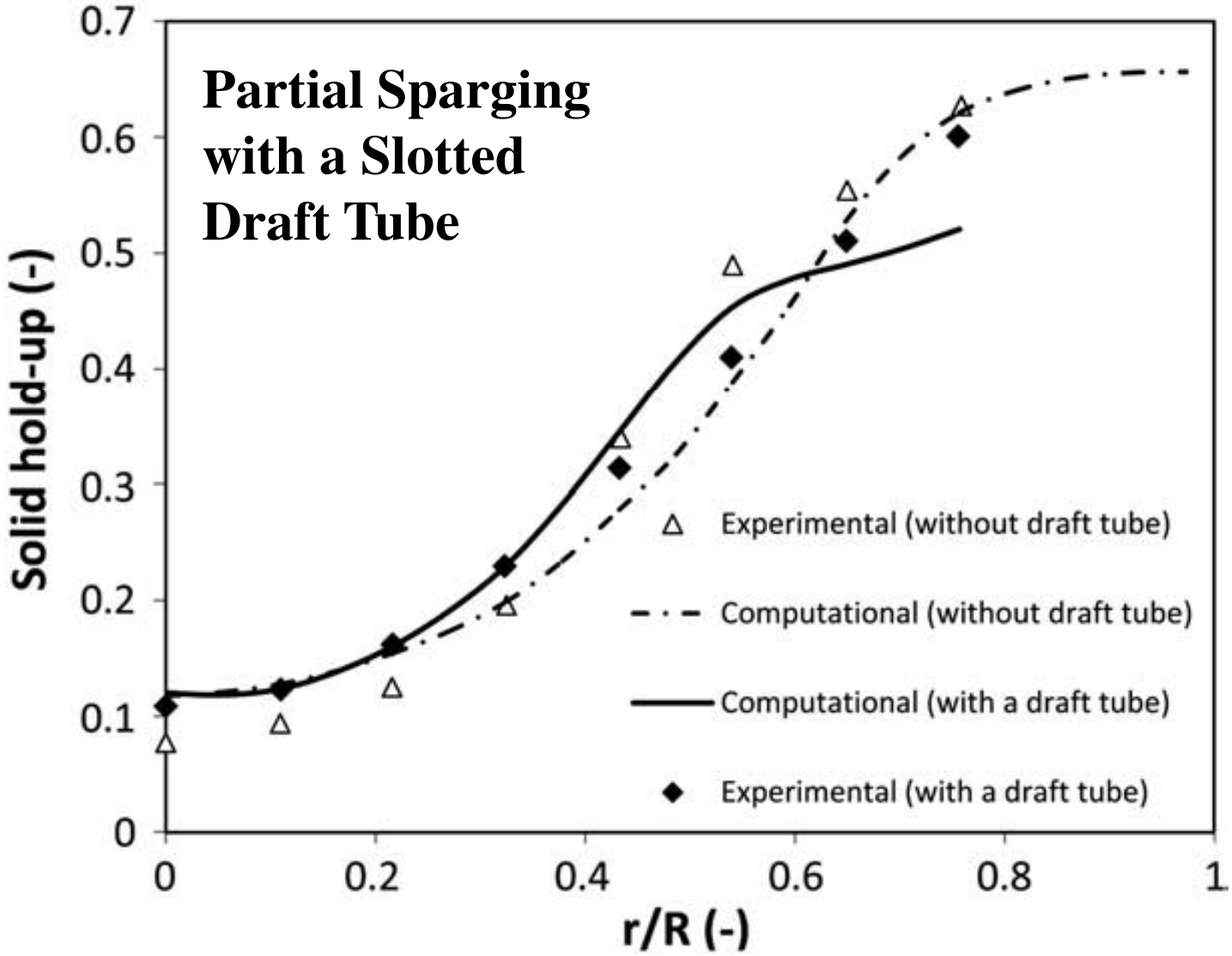
h/H=0.25

Partial Sparging



Solid Volume Fraction

Clarkson University



Conclusions

Clarkson University

- A two-fluid Eulerian model integrating the kinetic theory of granular flow and the k-ε turbulence model was used.
- Different gas sparging are investigated.
- The solid volume fraction profiles were predicted and compared with the experimental data.
- The model predictions agreed well with the experimental data.

Hosseini, Ahmadi, Rahimi, Zivdar, Nasr (2010)

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journal homepage: www.elsevier.com/locate/jtice

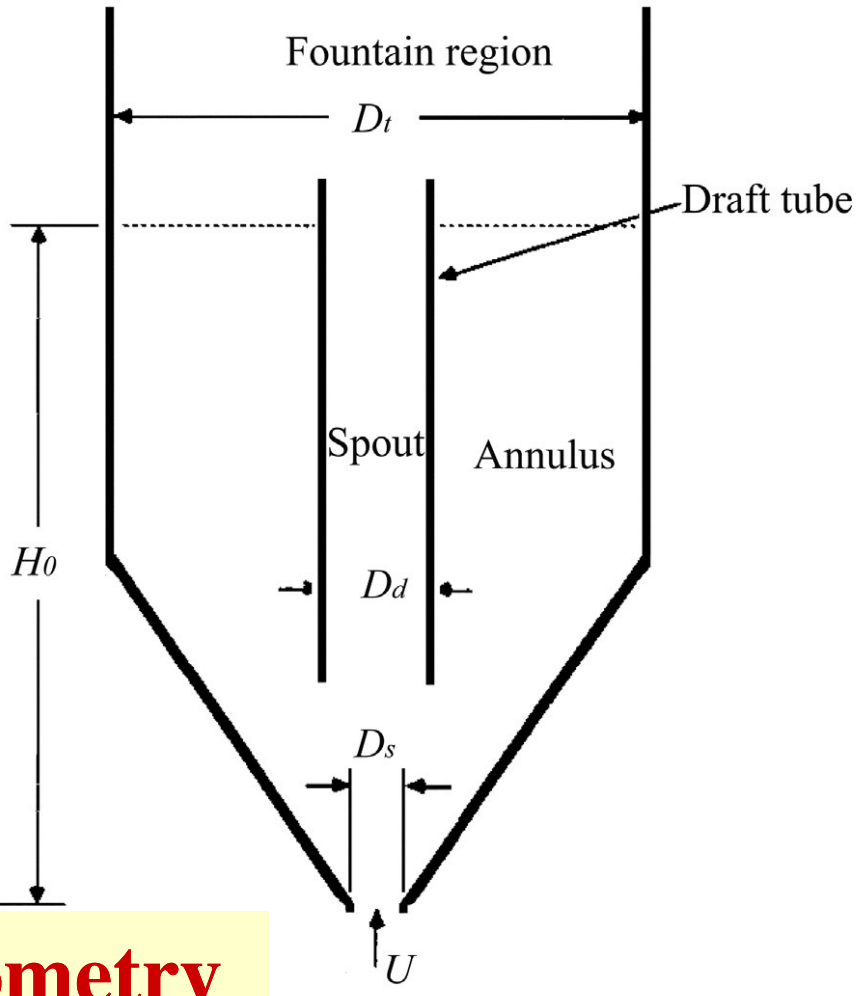
CFD study of particle velocity profiles inside a draft tube in a cylindrical spouted bed with conical base

Seyyed Hossein Hosseini [a,*], Goodarz Ahmadi [b], Martin Olazar [c]

Spout Bed Reactor

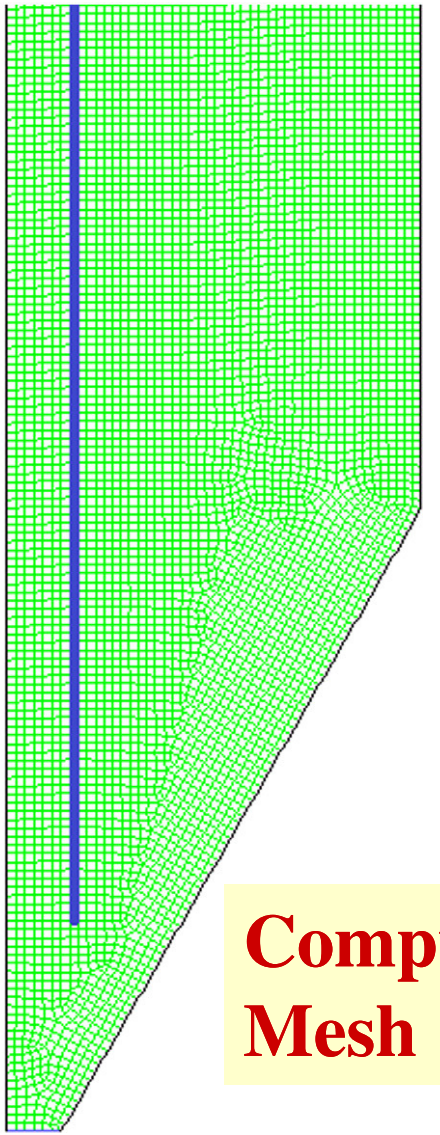
Clarkson University

(a)



Geometry

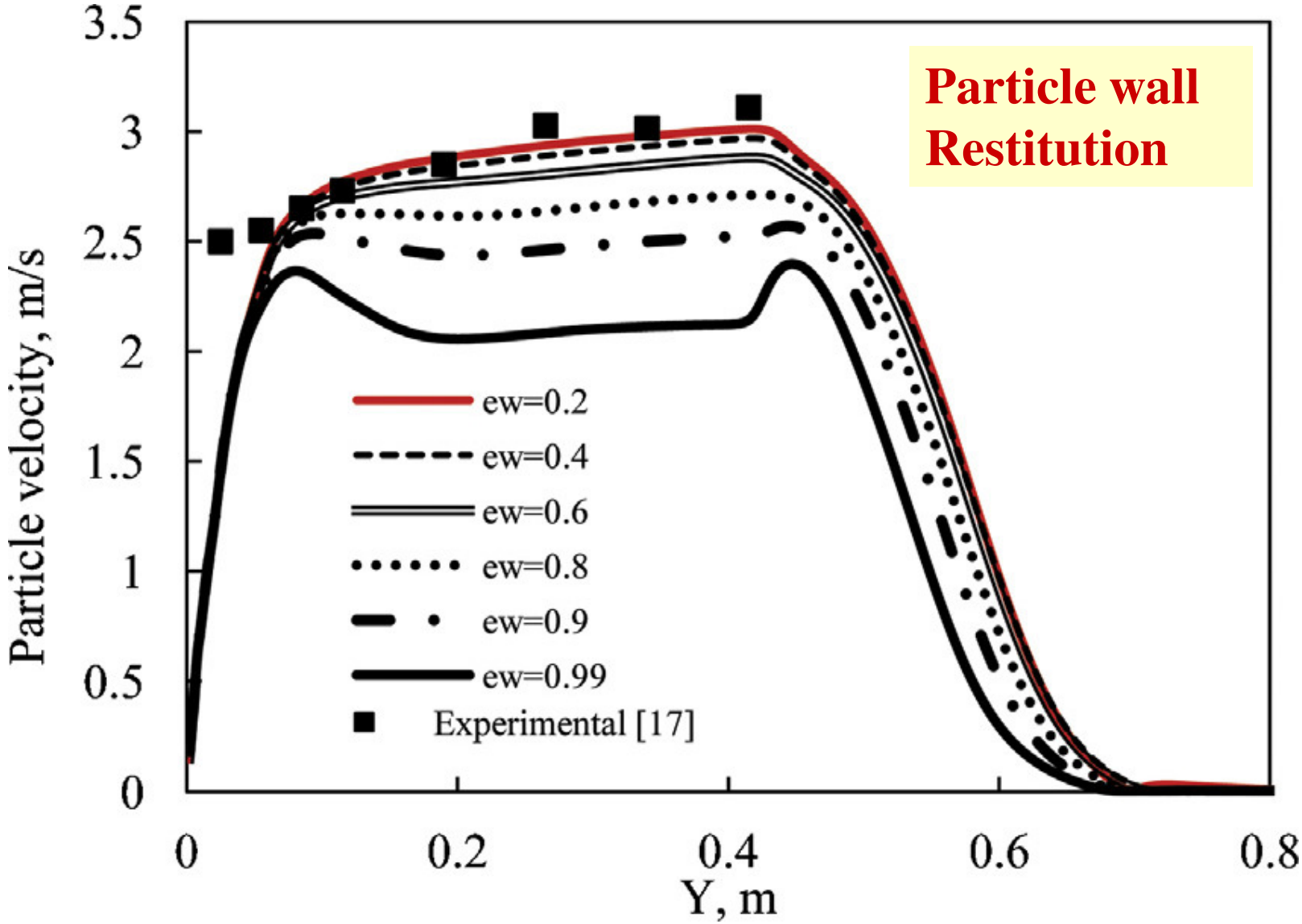
(b)



Computational Mesh

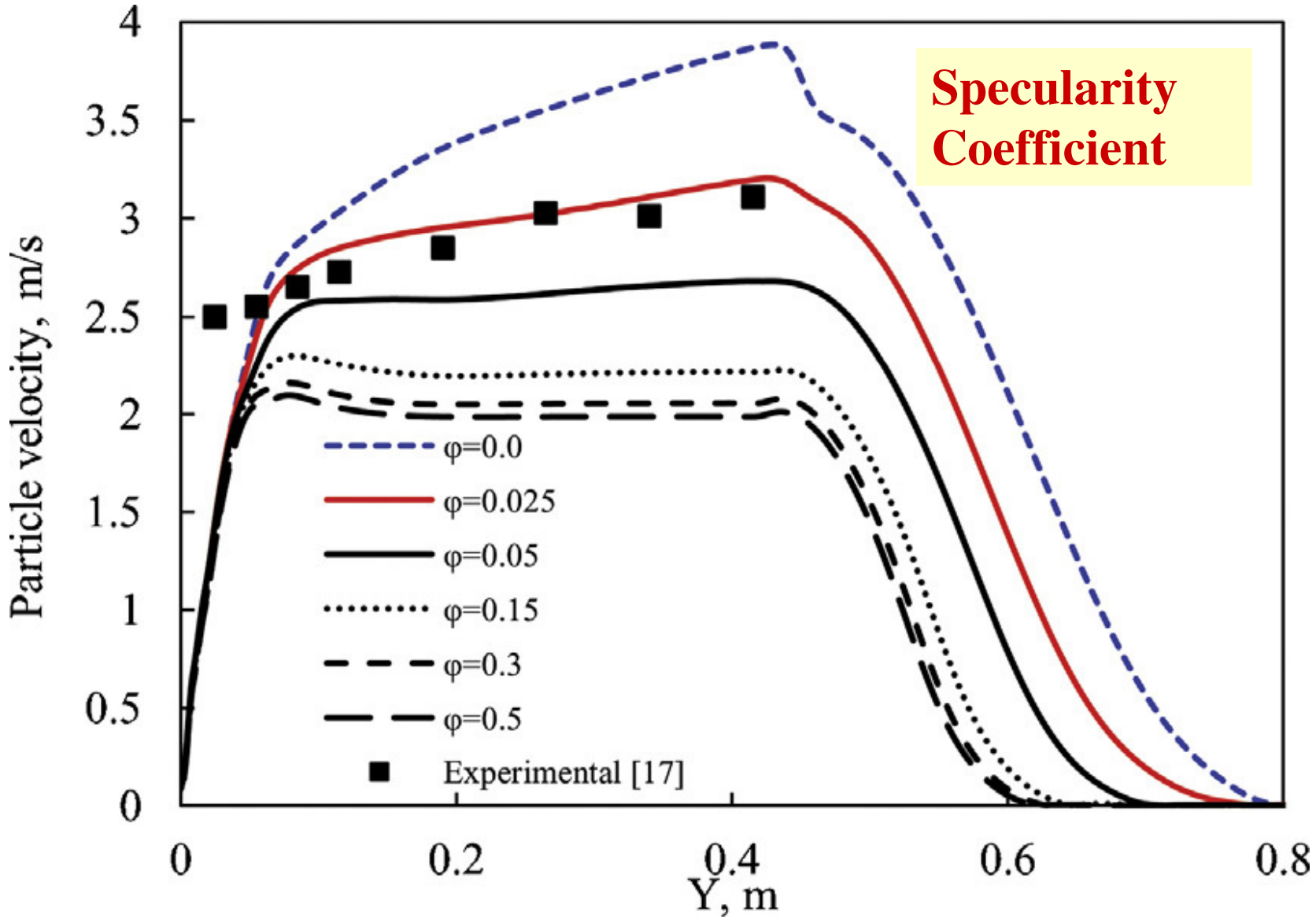
Axial Particle Velocity

Clarkson University



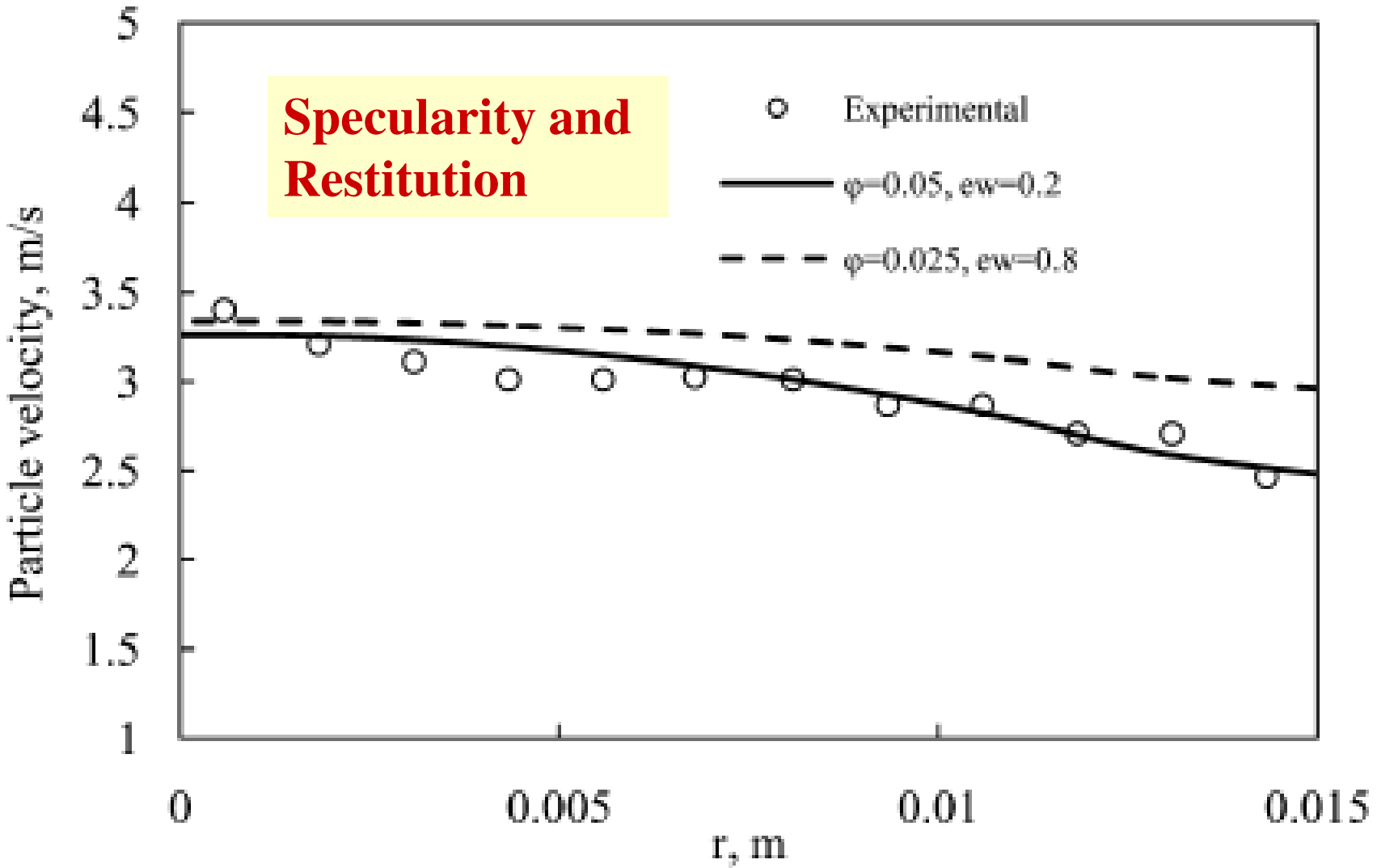
Axial Particle Velocity

Clarkson University



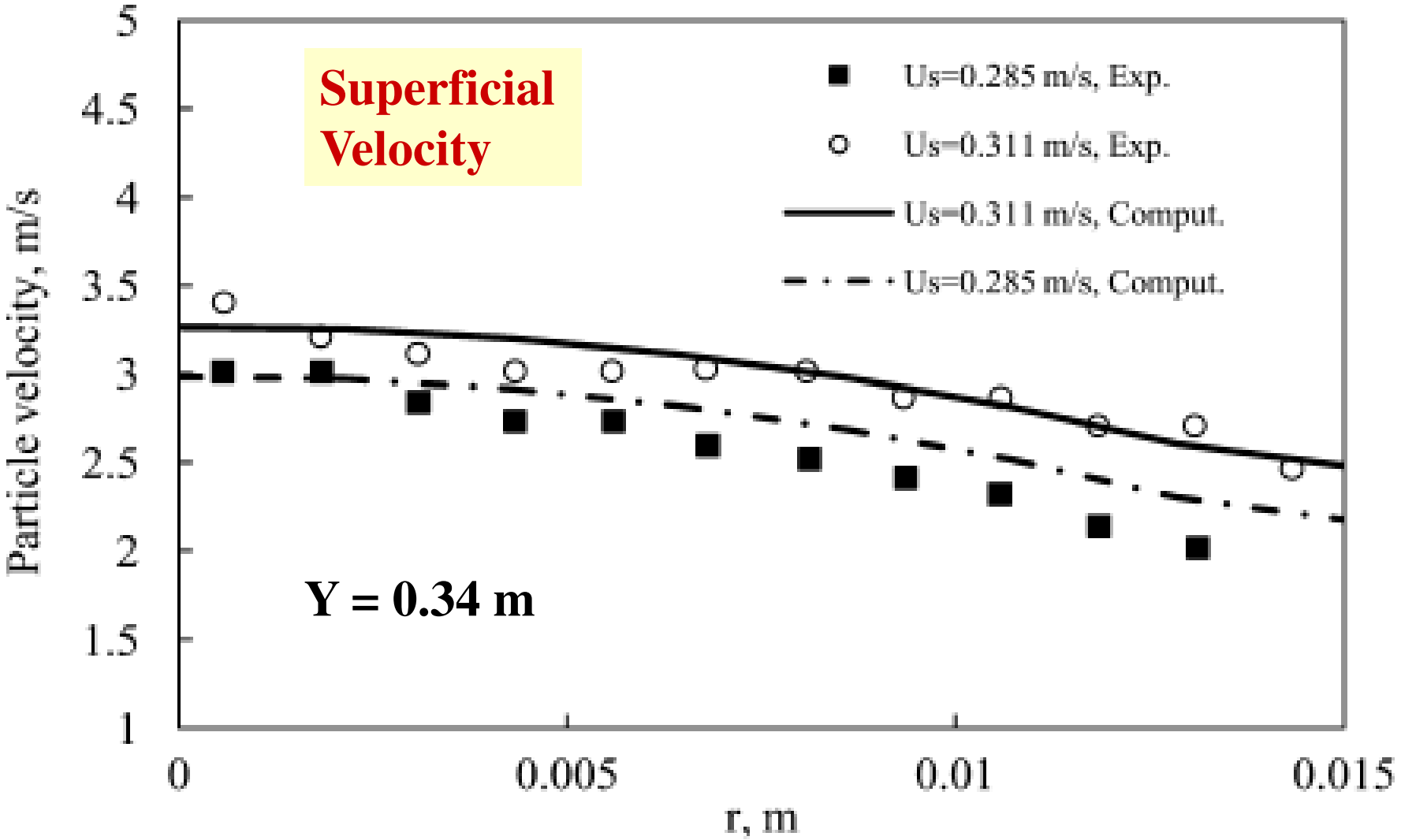
Particle Velocity Profile

Clarkson University



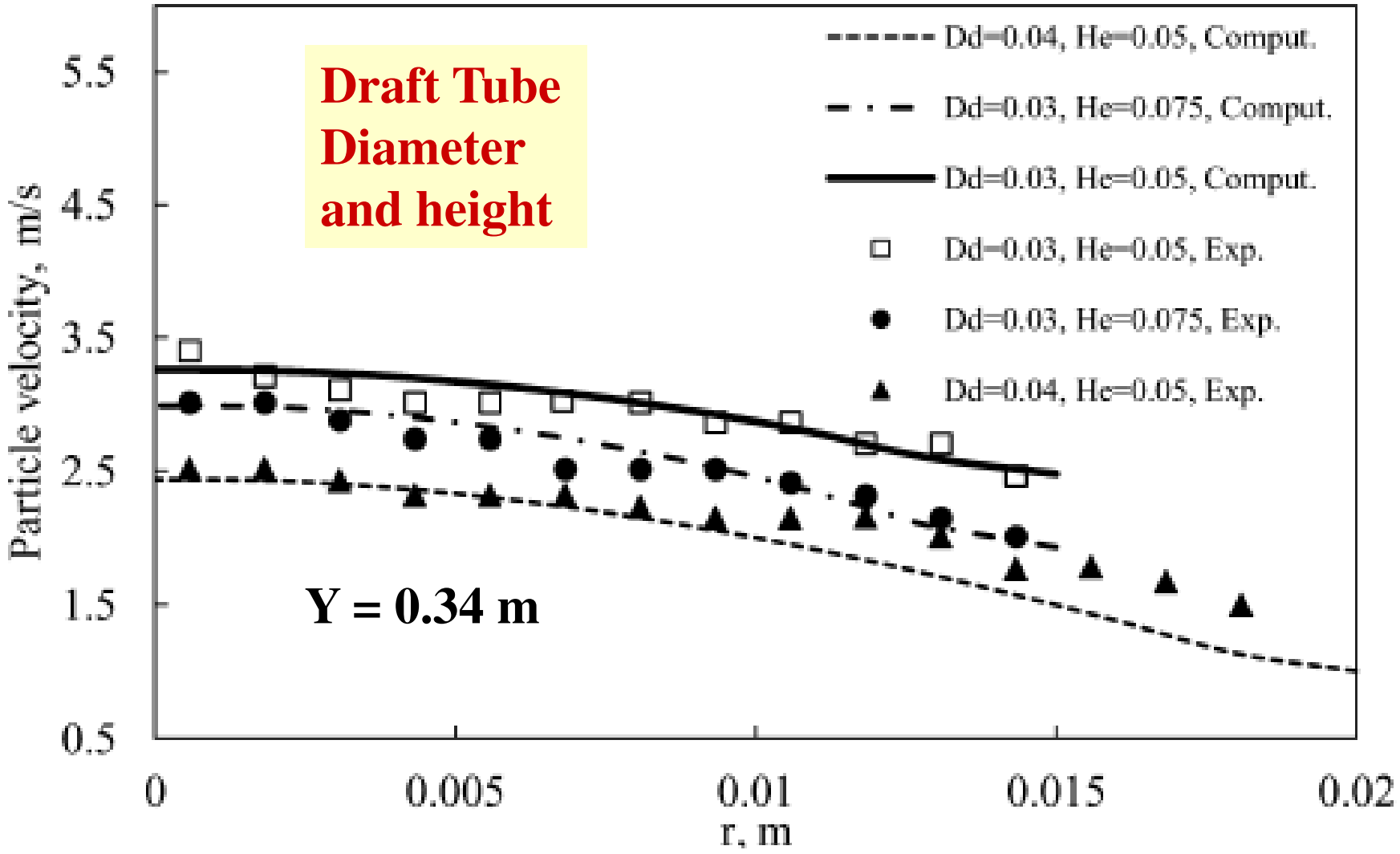
Particle Velocity Profile

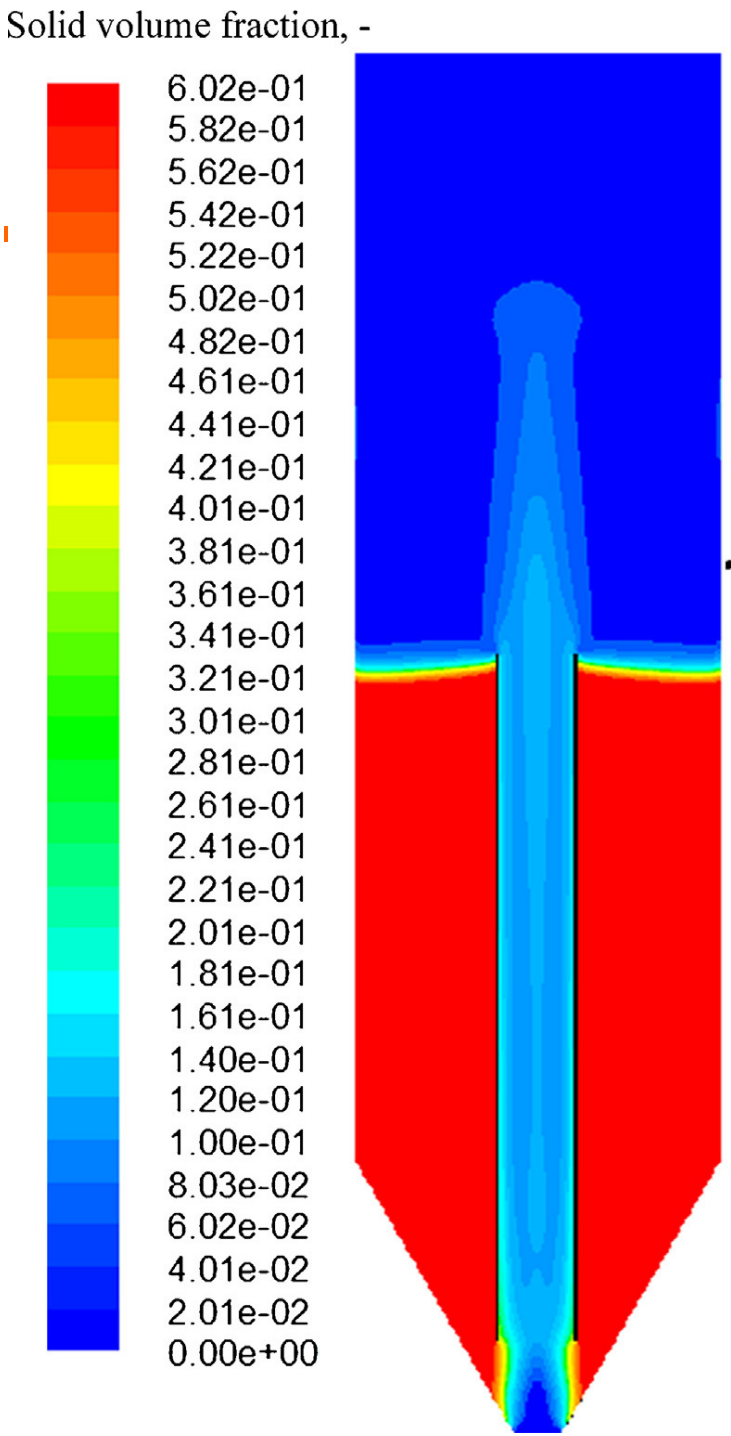
Clarkson University

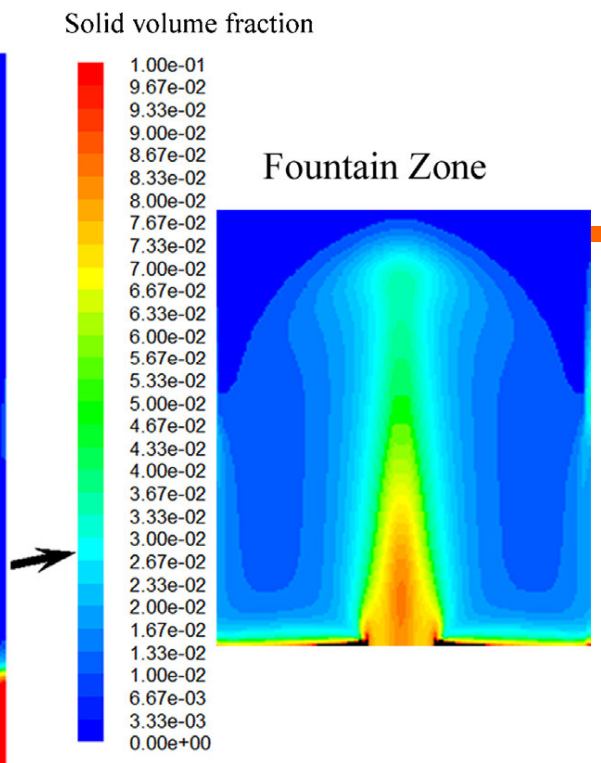


Particle Velocity Profile

Clarkson University







Fountain Zone

Clarkson University

Us = 0.285 m/s
Dd = 0.04 m.

Solid Volume Fraction

Conclusions

Clarkson University

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Hosseini, Ahmadi, Rahimi, Zivdar, Nasr (2010)

Clarkson University

Thank You!

Questions?