ICM 2014

International Workshop on Computational Mathematics - Advances in Computational PDEs Book of Abstracts

ICM 2014 Satellite Conference

Yonsei University, Korea, August 2014

Yonsei Campus Map

* Conference Venue : Yonsei-Samsung Library

Yonsei-Samsung Library entrance

International Workshop on Computational Mathematics -Advances in Computational PDEs

August 9-12, 2014

Chang Ki-Won Seminar room (Yonsei-Samsung Library 7F), Yonsei University

August 9 (Saturday)	
09:00 - 09:20	Registration*
09:20 - 09:40	Opening Ceremony : Eun-Jae Park
	Chair : Eun-Jae Park
09:40 - 10:30	Susanne Brenner (Louisiana State University, USA) Multigrid Methods for Saddle Point Problems
10:30 - 10:40	Break
	Chair : Carsten Carstensen
10:40 - 11:20	Hyung-Chun Lee (Ajou University, Korea) An Efficient Sparse Grid Stochastic Collocation Method for Stochastic Burgers Equation
11:20 – 12:00	Todd Arbogast (University of Texas-Austin, USA) Approximation of a Linear Degenerate Elliptic Equation Arising from A Two-Phase Mixture
12:00 - 14:00	Lunch break
	Chair : Todd Arbogast
14:00 - 14:50	Carsten Carstensen (Humboldt-University, Germany) Adaptive Eigenvalue Computation
14:50 - 15:30	Youngmi Hur (Yonsei University, Korea) Introduction to Multi-D Wavelet Constructions
15:30 - 16:00	Dongwook Shin (Yonsei University, Korea) Adaptive hybrid discontinuous Galerkin method for Poisson problem
16:00 - 16:30	Coffee break
	Chair : Eunjung Lee
16:30 - 17:10	Thanh Tran (The University of New South Wales, Australia) Transmission problems with random interfaces : A shape calculus approach
17:10 - 17:50	Kwang-Yeon Kim (Kangwon National University, Korea) Asymptotic Exactness of a Posteriori Error Estimators for the Lowest-Order Raviart--Thomas Mixed Finite Element Method

* Registration and all lectures will take place at Chang Ki-Won international conference room (Yonsei Library 7F).

August 10 (Sunday)

Open Discussion

August 11 (Monday)

	Chair : Zhiming Chen
09:00 - 09:50	Qiang Du (Columbia University, USA) Numerical Approximations of Nonlocal Diffusion and Peridynamic Models
09:50 - 10:30	Dongwoo Sheen (Seoul National University, Korea) TBA
10:30 - 10:50	Break
	Chair : Qiang Du
10:50 - 11:30	Ivan Yotov (University of Pittsburgh, USA) Domain Decomposition Methods for Coupled Flow and Mechanics Problems
11:30 - 12:10	Mi-Young Kim (Inha University, Korea) A discontinuous Galerkin method with Lagrange multiplier
12:10 - 14:00	Lunch break
	Chair : Ivan Yotov
14:00 - 14:40	Zhiming Chen (Chinese Academy of Sciences, China) The Pml Method and Fast Solvers for The Helmholtz Equation in Unbounded Domain
14:40 - 15:10	Seokchan Kim (Changwon National University, Korea) FEM for PDE With Domain Singularities – Dual Singular Function Method –
15:10 - 15:50	Eric Chung (The Chinese University of Hong Kong, Hong Kong) Staggered Discontinuous Galerkin Methods for Wave Propagation
15:50 - 16:20	Coffee break
	Chair : Eric Chung
16:20 - 17:00	Shuyu Sun (Kaust, Saudi Arabia) Discontinuous Galerkin for Subsurface Flow – Local Conservation, Algorithm Compatibility and Multiscale Coupling
17:00 - 17:30	Irene Sonja Monnesland (Yonsei University, Korea) Least squares finite element method for a nonlinear problem in glaciology
18:00 - 20:00	Banquet

August 12 (Tuesday)	
	Chair : Young-Ju Lee
09:00 - 09:50	Alexandre Ern (University Paris-Est, France) Hybrid high-order schemes on general meshes for elliptic PDEs
09:50 - 10:30	Xiao-Chuan Cai (University of Colorado-Boulder, USA) Space-Time Schwarz Methods for Solving Inverse Problems in 4D
10:30 - 10:50	Break
	Chair : Xiao-Chuan Cai
10:50 - 11:30	Young-Ju Lee (Texas State University, USA) Stable Hybrid Discretizations and Fast Solvers for Non-Newtonian Fluids Flows and Applications
11:30 - 12:10	Xiaoming He (Missouri S&T, USA) A Multi-Physics Domain Decomposition Method for Navier-Stokes-Darcy Model
12:10 - 14:00	Lunch break
	Chair : Xiaoming He
14:00 - 14:40	Youngmok Jeon (Ajou University, Korea) The hybrid FEM and FDM for PDEs
14:40 - 15:20	Viet Ha Hoang (Nanyang Technological University, Singapore) An Efficient Hierachical Multiscale Finite Element Method for Stokes Equations in Slowly Varying Media
15:20 - 15:50	Xuefeng Liu (Waseda University, Japan) A Uniform Framework to Provide Guaranteed Eigenvalue Bounds for Self-Adjoint Differential Operators
15:50 - 16:30	Amiya Pani (IIT-Bombay, India) Superconvergent DGM for second order elliptic boundary value problems

Day 1

MULTIGRID METHODS FOR SADDLE POINT PROBLEMS

SUSANNE C. BRENNER

Abstract. In this talk we will present a general framework for the design and analysis of multigrid methods for saddle point problems arising from mixed finite element discretizations of elliptic boundary value problems. These multigrid methods are uniformly convergent in the energy norm on general polyhedral domains where the elliptic boundary value problems in general do not have full elliptic regularity. Applications to Stokes, Lamé, Darcy and related nonsymmetric systems will be discussed. This is joint work with Hengguang Li, Duk-Soon Oh and Li-Yeng Sung.

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Key words and phrases. Saddle Point Problems, Mixed Finite Element Methods, Multigrid Methods.

AN EFFICIENT SPARSE GRID STOCHASTIC COLLOCATION METHOD FOR STOCHASTIC BURGERS EQUATION

HYUNG-CHUN LEE* AND YUN NAM

Abstract. We describe an efficient approximation of solution to stochastic Burgers equation driven by an additive space-time noise. We discuss existence and uniqueness of a solution through the Orstein-Uhlenbeck (OU) process. To approximate the OU process, we introduce the Karhunen-Loève expansion, and sparse grid stochastic collocation method. About spatial discretization of Burgers equation, two separate finite element approximations are presented: the conventional Galerkin method and Galerkin-conservation method. Numerical experiments are provided to demonstrate the efficacy of schemes mentioned above.

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Key words and phrases. Burgers' equation, Orstein-Uhlenbeck process, Karhunen-Loève expansion, Sparse grids collocation, Finite element method.

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APPROXIMATION OF A LINEAR DEGENERATE ELLIPTIC EQUATION ARISING FROM A TWO-PHASE MIXTURE

TODD ARBOGAST, MARC A. HESSE, AND ABRAHAM L. TAICHER

ABSTRACT. We consider the linear but degenerate elliptic system of two first order equations $\mathbf{u} = -\phi\nabla p$ and $\nabla\cdot\mathbf{u} + \phi p = \phi f$, where the *porosity* $\phi \geq 0$ may be zero on a set of positive measure. The model equation we consider has a similar degeneracy as that arising in the equations describing the mechanical system modeling the dynamics of partially melted materials, e.g., in the Earth's mantle, and the flow of ice sheets, e.g., in the polar ice caps and glaciers. In the context of mixture theory, ϕ represents the phase variable separating the solid one-phase ($\phi = 0$) and fluid-solid two phase ($\phi > 0$) regions. Two main problems arise. First, as ϕ vanishes, one equation is lost. Second, after we extract stability or energy bounds for the solution, we see that the *pressure* p is not controlled outside the support of ϕ. After an appropriate scaling of the pressure, we can show existence and uniqueness of a solution over the entire domain. We then develop a stable mixed finite element method for the problem, and show some numerical results.

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Key words and phrases. Degenerate Elliptic, Mixture Theory, Energy Bounds, Ice Sheets, Mantle Dynamics, Mixed Method.

ADAPTIVE EIGENVALUE COMPUTATION

CARSTEN CARSTENSEN

Abstract. This talk presents recent advances in the nonconforming FEM approximation of elliptic PDE eigenvalue problems. The first part introduces guaranteed lower eigenvalue bounds for second-order and fourth-order eigenvalue problems with relevant applications for the localization of in the critical load in the buckling analysis of the Kirchhoff plates. The second studies an optimal adaptive mesh-refining algorithm for the effective eigenvalue computation for the Laplace operator with optimal convergence rates in terms of the number of degrees of freedom relative to the concept of nonlinear approximation classes. The analysis includes an inexact algebraic eigenvalue computation on each level of the adaptive algorithm which requires an iterative algorithm and a controlled termination criterion. The third part extends the analysis to multiple and even clustered eigenvalues. The topics reflect joint work with Dr. Joscha Gedike (LSU) and Dr Dietmar Gallistl (Bonn).

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INTRODUCTION TO MULTI-D WAVELET CONSTRUCTIONS

YOUNGMI HUR

Abstract. Wavelets are introduced as an alternative to the Fourier representation around late 80s. Since their introduction, wavelets have been used in many applications including signal and image processing, among other areas. In this talk, we first review wavelets, especially with the focus of how to construct them in multidimensional setting. We then present some new multidimensional wavelet construction methods that are developed recently.

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ADAPTIVE HYBRID DISCONTINUOUS GALERKIN METHOD FOR POISSON PROBLEM

EUN-JAE PARK AND DONG-WOOK SHIN

Abstract. A hybrid discontinuous Galerkin method was introduced by Jeon and Park. The method has attractive properties. First, the method allows high-order polynomial approximations and it is easy to implement. Second, the local conservation property holds. Third, the average flux is continuous across the interface boundary for even-degree polynomial approximations. Lastly, the global degrees of freedom are only on the skeleton. So global degrees of freedom can be reduced when $k \geq 3$ as compared with continuous Galerkin method's. In this work, an adaptive algorithm is applied to the hybrid discontinuous Galerkin method. Our a posteriori error estimator yields reliable and efficient estimates. And fully computable error estiator is obtained by using even-degree polynomial approximations. For the estimator, we construct the potential reconstruction with averaging operator. Several numerical examples are presented to illustrate the performance of the proposed estimator.

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Key words and phrases. adaptive algorithm, a posteriori error estimator, hybridization, discontinuous Galerkin.

TRANSMISSION PROBLEMS WITH RANDOM INTERFACES: A SHAPE CALCULUS APPROACH

ALEXEY CHERNOV, DUONG PHAM, AND THANH TRAN

Abstract. In this talk we will present an approximation to the statistical moments of the solution of a class of elliptic transmission problems in $\mathbb{R}^3$ with uncertainly located transmission interfaces. In this model, the diffusion coefficient has a jump discontinuity across the random transmission interface which models linear diffusion in two different media separated by an uncertain surface. We apply shape calculus to approximate the solution perturbation by the so-called *shape derivative*. Correspondingly statistical moments of the solution are approximated by the moments of the shape derivative. We characterize the shape derivative as a solution of a related homogeneous transmission problem with nonzero jump conditions, which is solved by the boundary integral equation method.

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Key words and phrases. transmission problem, random interface, shape calculus, shape derivative, boundary integral equation method.

ASYMPTOTIC EXACTNESS OF A POSTERIORI ERROR ESTIMATORS FOR THE LOWEST-ORDER RAVIART–THOMAS MIXED FINITE ELEMENT METHOD

KWANG-YEON KIM

Abstract. A posteriori error estimation has been an effective tool in guiding adaptive mesh refinement by estimating the generally unknown numerical error in terms of the numerical solution and the given data. An error estimator η should be reliable and efficient in the sense that the ratio of η to the numerical error e measured in some norm $\|\!|\cdot|\!\|$ (usually called the effectivity index) should remain bounded above and below like

$$C_1\eta + (\text{h.o.t}) \le \|\!|e|\!\| \le C_2\eta + (\text{h.o.t}),$$

where C_i's are positive constants independent of the mesh size h and (h.o.t) stands for higher order terms. In particular, we say that η is asymptotically exact if $C_1 = C_2 = 1$, i.e.,

$$\|\!|e|\!\| = \eta + (\text{h.o.t}) \qquad \text{or} \qquad \lim_{h\to 0} \frac{\eta}{\|\!|e|\!\|} = 1.$$

This implies that η can be very accurate as the mesh is refined (but under some favorable conditions). In this talk we discuss two such error estimators for the lowest-order Raviart–Thomas mixed finite element method on triangular meshes. One is an estimator of the Bank–Weiser type based on solution of local problems and was introduced in [Alonso, Error estimators for a mixed method, Numer. Math. 74 (1996), 385–395]. The other is of the hierarchical type and is based on solution of a global defect problem. The latter estimator was introduced as a part of the hierarchical error estimator in [Wohlmuth and Hoppe, A comparison of a posteriori error estimators for mixed finite element discretizations by Raviart–Thomas elements, Math. Comp. 68 (1999), 347–1378]. Asymptotic exactness of both error estimators is proved by using the super-closeness between the finite element solution and the Fortin projection of the exact solution.

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Key words and phrases. A Posteriori Error Estimator, Asymptotic Exactness, Raviart–Thomas Mixed Finite Element Method.

Day 2

Open Discussion

Day 3

NUMERICAL APPROXIMATIONS OF NONLOCAL DIFFUSION AND PERIDYNAMIC MODELS

QIANG DU

Abstract. Nonlocality is ubiquitous in nature. While partial differential equations (PDE) have been used as effective models of many physical processes, nonlocal models and nonlocal balanced laws are also attracting more and more attentions as possible alternatives to treat anomalous process and singular behavior. In this talk, we exploit the use of a recently developed nonlocal vector calculus to study a class of constrained value problems on bounded domains associated with some nonlocal balance laws. The nonlocal calculus of variations then offers striking analogies between nonlocal model and classical local PDE models as well as the notion of local and nonlocal fluxes. We discuss the consistency of nonlocal models to local PDE limits as the horizon, which measures the range of nonlocal interactions, approaches zero. In addition, we present asymptotically compatible discretizations that provide convergent approximations in the nonlocal setting with a nonzero horizon and are also convergent asymptotically to the local limit as both the horizon and the mesh size are taking to zero. Such asymptotically compatible discretizations can be more robust for multiscale problems with varying length scales.

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Key words and phrases. Computational Mathematics, Differential and Integral Equations, Finite Element Methods.

DOMAIN DECOMPOSITION METHODS FOR COUPLED FLOW AND MECHANICS PROBLEMS

IVAN YOTOV

Abstract. We discuss a computational framework for modeling multiphysics systems of coupled flow and mechanics problems. The approach is based on a multiblock domain decomposition methodology. The simulation domain is decomposed into a union of subdomains, each one associated with a physical, mathematical, and numerical model. Physically meaningful interface conditions are imposed on the discrete level via mortar finite elements or Nitsche's coupling. We present two applications of the framework: 1) multiscale mortar discretizations of coupled Stokes-Darcy flows and 2) Nitsche's method for Stokes/Brinkman flows coupled with the Biot system of poroelasticity. We discuss stability and accuracy of the spatial discretizations and loosely coupled non-iterative time split formulations. We further study the use of the loosely coupled scheme as a preconditioner for the monolithic scheme and establish a spectral equivalence of the two formulations. Applications to coupled surface and groundwater flows, flows in fractured deformable reservoirs, and arterial flows are presented.

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Key words and phrases. domain decomposition, mortar finite element, mltiphysics, multiscale.

A DISCONTINUOUS GALERKIN METHOD WITH LAGRANGE MULTIPLIER

MI-YOUNG KIM

Abstract. We develop a discontinuous Galerkin method with Lagrange multiplier (DGLM) to approximate the solution of the second-order elliptic problems. Lagrange multipliers are introduced on the edge/face of the element. A Dirichlet boundary condition is weakly imposed through the edge/face of the element. The continuity of the solution is weakly imposed through the Lagrange multiplier space. The DGLM *localizes* the approximation. The global system of Lagrange multipliers has substantially fewer numbers of unknowns than the standard discontinuous Galerkin (DG) methods. The diffusion coefficient is allowed to degenerate. Stability of the approximate solution is proved. Local error estimates of optimal convergence rates are derived. It is shown that the method preserves the local mass conservation. An explanation on algorithmic aspects is given. Extensions of the DGLM to nonlinear conservation laws will be also addressed.

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Key words and phrases. Discontinuous Galerkin method, Partial Differential Equations, Finite Element Methods.

THE PML METHOD AND FAST SOLVERS FOR THE HELMHOLTZ EQUATION IN UNBOUNDED DOMAIN

ZHIMING CHEN

Abstract. Wave scattering is ubiquitous in modern scientific and engineering applications. One of the fundamental problems in the efficient simulation of wave scattering phenomena is the reduction of the exterior problem which is defined in the unbounded domain to the problem in the bounded domain. We shall introduce the idea of perfectly matched layer (PML) for truncating wave scattering problems and report our progress in designing fast solvers for large wave number discrete Helmholtz equations using the idea of PML method. This talk is based on joint works with Xueshuang Xiang.

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FEM FOR PDE WITH DOMAIN SINGULARITIES – DUAL SINGULAR FUNCTION METHOD –

SEOKCHAN KIM

Abstract. In [Z. Cai, S. Kim, A finite element method using singular functions for the Poisson equation: corner singularities, SIAM J. Numer. Anal. 39 (2001) 286.299], we proposed a new finite element method to compute singular solutions of Poisson equations on a polygonal domain with re-entrant angles. Singularities are eliminated and only the regular part of the solution that is in H^2 is computed. The stress intensity factor and the solution can be computed as a post-processing step. This method was extended to several problems; 1) crack singularities 2) mixed boundary conditions 3) interface problems.

Now this talk is focused in finding i) the modification of this approach and ii) its extension to linear elasticity problem.

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Key words and phrases. Computational Mathematics, Partial Differential Equations, Finite Element Methods.

STAGGERED DISCONTINUOUS GALERKIN METHODS FOR WAVE PROPAGATION

ERIC CHUNG

Abstract. A new class of discontinuous Galerkin method is recently developed for wave propagation. The method is based on a staggered grid and a pair of discontinuous Galerkin spaces defined on it. The resulting method is energy conserving with optimal convergence and block diagonal mass matrix. In addition, the dispersion error is two order higher than non-staggered methods. Thus, the staggered discontinuous Galerkin methods provide a convincing alternative to existing methodologies. In this talk, we will present the method for acoustic and electromagnetic wave propagations. We will also briefly overview some applications to other problems.

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Key words and phrases. Computational Mathematics, Partial Differential Equations, Finite Element Methods.

DISCONTINUOUS GALERKIN FOR SUBSURFACE FLOW – LOCAL CONSERVATION, ALGORITHM COMPATIBILITY AND MULTISCALE COUPLING

SHUYU SUN

Abstract. Subsurface multiphase flow has important applications in many areas, particularly in oil-gas industry and subsurface environmental protection. Modeling and simulation of subsurface flow has been conducted for decades, but mainly using the lowest order finite difference schemes in practice. In recent years, discontinuous Galerkin (DG) methods start to get closed attention in this area. The DG methods are specialized finite element methods to approximate the solutions of differential equations using discontinuous piecewise polynomial spaces, with boundary conditions and inter-element continuity weakly imposed through bilinear forms. Derived from variational principles by integration over local cells, the methods are locally mass conservative by construction. Weak enforcement of boundary conditions and inter-element continuity leads to small numerical diffusion and little oscillation for DG. Moreover, they have excellent parallel efficiency since data communications are relatively local. In addition, the DG methods handle rough coefficient problems and capture the discontinuity in the solution very well by the nature of discontinuous function spaces. They support general nonconforming spaces including unstructured meshes, nonmatching grids and variable degrees of local approximations, thus allowing efficient h-, p-, and hp-adaptivities. In this talk, we apply Discontinuous Galerkin for the transport equations of compositional single-phase and two-phase flow in porous media. We also apply Discontinuous Galerkin for the immiscible two-phase Darcy flow, where at each time step we solve the pressure equation for (total) Darcy velocity followed by Discontinuous Galerkin solution of the saturation equation. We present a new scheme that conserves all species locally. This scheme, when applied to immiscible two-phase Darcy flow, conserves both phases locally. We propose a theory of algorithm compatibility for coupled physics, and we also present error analysis on DG, especially a priori error bounds and a posteriori error estimates in various norms. Efficient implementation issues are addressed with emphasis on dynamic mesh adaptation. A rich number of numerical examples are given to illustrate various features of DG methods including their small numerical diffusion, reduced mesh orientation effect, sharp a posteriori error indicators, and effective mesh adaptation. Finally, we discuss the possible multiscale coupling of compositional flow between the Darcy scale and the pore scale.

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Key words and phrases. Subsurface Flow and Transport, Porous Media Flow, Multiphase Flow, Algorithm Campatibility, Local Conservation, Multiscale Algorithm.

LEAST SQUARES FINITE ELEMENT METHOD FOR A NONLINEAR PROBLEM IN GLACIOLOGY

IRENE SONJA MONNESLAND

ABSTRACT. A nonlinear, incompressible Stokes problem arising glaciology is considered in this paper. In glaciology, the ice flows in glaciers and ice sheets so that the boundary condition at the ice-bedrock interface is often described by Coulomb-like friction. We use the Rayleigh friction boundary condition for sliding basal boundary. In order to find an approximation for nonlinear problem, we adopt the modified Picard iteration. Once linearized system is obtained, we present two different minimization problems and show the existence and uniqueness of corresponding minimizers. Finally we analyze the errors in finite element approximations and then show numerical some results.

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Day 4

HYBRID HIGH-ORDER SCHEMES ON GENERAL MESHES FOR ELLIPTIC PDES

DANIELE DI PIETRO AND ALEXANDRE ERN

Abstract. We develop and analyze a family of arbitrary-order, compact-stencil discretization schemes for elliptic PDEs on polyhedral meshes. The key idea is to reconstruct differential operators cell-wise in terms of the local degrees of freedom. Optimal error estimates for the flux and the potential are derived. Links with other recent approaches from the literature are discussed. The methodology is also applied to linear elasticity problems, leading to locking-free schemes. The theoretical results are confirmed numerically.

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Key words and phrases. Linear elasticity, general meshes, arbitrary order, locking-free methods.

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SPACE-TIME SCHWARZ METHODS FOR SOLVING INVERSE PROBLEMS IN 4D

XIAO-CHUAN CAI

Abstract. In this talk, we discuss some coupled space-time domain decomposition methods for solving unsteady inverse problems in four-dimensional spaces. Inverse source identification problems have important applications in many areas, such as air pollution source tracking, and are usually difficult to solve especially when both the location history and time history are of interests. We focus on some recently developed space-time overlapping Schwarz methods, and present some numerical experiments carried out on supercomputers with a large number of processors. This is a joint work with X. Deng and J. Zou.

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Key words and phrases. unsteady inverse problem, domain decomposition method, parallel scalability.

STABLE HYBRID DISCRETIZATIONS AND FAST SOLVERS FOR NON-NEWTONIAN FLUIDS FLOWS AND APPLICATIONS

YOUNG JU LEE

Abstract. We present a hybrid solver for general rate-type non-Newtonian fluids models. This method is fully implicit and each nonlinear iteration consists of three steps: (1) locating the characteristic feet of fluid particles, (2) solving the momentum equation and continuity equation, and (3) solving the constitutive equations. For stability and accuracy purposes, we employ a higher order conforming approximation of the Stokes equation on the target mesh and a lower order stress field approximation on a finer mesh. For the solution to the conforming finite element methods for the Stokes equation, we apply an auxiliary space preconditioning method, in which low-order finite element spaces are employed as auxiliary spaces. The stress equation is formulated on the refined grids to reduce the accuracy gap between velocity and stress. Numerical results demonstrate the advantages of the proposed algorithm in terms of efficiency, robustness, and weak scalability.

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Key words and phrases. Computational Mathematics, Partial Differential Equations, Finite Element Methods.

A MULTI-PHYSICS DOMAIN DECOMPOSITION METHOD FOR NAVIER-STOKES-DARCY MODEL

XIAOMING HE, JIAN LI, AND YANPING LIN

Abstract. The Navier-Stokes-Darcy model arises in many interesting real world applications, including groundwater flows in karst aquifers, interaction between surface and subsurface flows, industrial filtrations, fractured reservoir, and so on. This model describes the free flow of a liquid by the Navier-Stokes equation and the confined flow in a porous media by the Darcy equation; the two flows are coupled through interface conditions.

This presentation discusses a multi-physics domain decomposition method for solving the coupled steady state Navier-Stokes-Darcy system with the Beavers-Joseph interface condition. The wellposedness of this system is first showed by using a branch of singular solutions and the existing theoretical results on the Beavers-Joseph interface condition. Then Robin boundary conditions on the interface are constructed based on the physical interface conditions to decouple the Navier-Stokes and Darcy parts of the system. A parallel iterative domain decomposition method is developed according to these Robin boundary conditions and then analyzed for the convergence. Numerical examples are presented to illustrate the features of this method and verify the theoretical results.

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Key words and phrases. Multi-physics domain decomposition, Navier-Stokes-Darcy model, Beavers-Joseph interface condition.

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THE HYBRID FEM AND FDM FOR PDES

YOUNGMOK JEON

Abstract. In this presentation we introduce the hybrid FEM and FDM. The methods are base on 1) constructing local solutions on each triangle of a triangulation 2) patching local solutions by using the interface condition on intercell boundaries. The main advantages of our approaches are that

1. They are locally conservative
2. There is big reduction reduction in degrees of freedom, compared to the conventional finite element methods
3. When applied to flow problems, the degrees of freedom are reduced to the skeleton data of the velocity vector filed and the cell average of the pressure.

In this talk we rather focus on the hybrid FDM for the Navier-Stokes equations. The hybrid FDM possesses interesting features compared to the existing FDM

1. It is easy to implement since the primal variables are approximated on the collocated grids (without a staggered grid).
2. The method retains the optimal order of convergence even with non-uniform grids.
3. The method can be modified to accommodate problems with curved boundaries.

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Key words and phrases. hybrid, FEM, FDM, Navier-Stokes, Stokes.

AN EFFICIENT HIERACHICAL MULTISCALE FINITE ELEMENT METHOD FOR STOKES EQUATIONS IN SLOWLY VARYING MEDIA

VIET HA HOANG

Abstract. Direct numerical simulation of fluid flow in porous media with many scales is often not feasible, and an effective or homogenized description is more desirable. To construct the homogenized equations, effective properties must be computed. Computation of effective properties for nonperiodic microstructures can be prohibitively expensive, as many local cell problems must be solved for different macroscopic points. In addition, the local problems may also be computationally expensive. When the microstructure varies slowly, we develop an efficient numerical method for two scales that achieves essentially the same accuracy as that for the full resolution solve of every local cell problem. In this method, we build a dense hierarchy of macroscopic grid points and a corresponding nested sequence of approximation spaces. Essentially, solutions computed in high accuracy approximation spaces at select points in the the hierarchy are used as corrections for the error of the lower accuracy approximation spaces at nearby macroscopic points. We give a brief overview of slowly varying media and formal Stokes homogenization in such domains. We present a general outline of the algorithm and list reasonable and easily verifiable assumptions on the PDEs, geometry, and approximation spaces. With these assumptions, we achieve the same accuracy as the full solve. To demonstrate the elements of the proof of the error estimate, we use a hierarchy of macrogrid points in $[0,1]^2$ and finite element approximation spaces in $[0,1]^2$. We apply this algorithm to Stokes equations in a slowly porous medium where the microstructure is obtained from a reference periodic domain by a known smooth map. Using the arbitrary Lagrange-Eulerian formulation of the Stokes equations we obtain modified Stokes equations with varying coefficients in the periodic domain. We implement the algorithm on the modified Stokes equations, using a simple stretch deformation mapping, and compute the effective permeability. We show that our efficient computation is of the same order as the full solve.

This is a joint work with Donald Brown (University of Bonn) and Yalchin Efendiev (Texas A&M University).

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A UNIFORM FRAMEWORK TO PROVIDE GUARANTEED EIGENVALUE BOUNDS FOR SELF-ADJOINT DIFFERENTIAL OPERATORS

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Abstract. In this talk, a uniform framework to provide guaranteed eigenvalue bounds for self-adjoint differential operators is proposed. In this framework, both conforming and non-conforming finite element methods (FEMs) are adopted to construct explicit eigenvalue bounds, even in the case that the eigenfunction has a singularity around the re-entrant corners of domains. As concrete examples, the conforming Lagrange finite element is used to give eigenvalue bounds for the Laplacian defined over polygonal domains of general shapes, where the technique of hypercircle equation from Prager-Synge's theorem plays an important role. The Crouzeix-Raviart FEMs and the Fujino-Morley FEMs, along with explicit apriori error estimation, are used to provide explicit eigenvalue bounds for the Laplacian and the Bi-harmonic operators, respectively. Further, Lehmann-Goerisch's theorem is applied to give dramaticly improved high-precision eigenvalue bounds. As the computation is performed under the interval arithmetic, the obtained eigenvalue bounds are mathematically correct and thus can be used in solution existence verification for certain non-linear partial differential equations.

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SUPERCONVERGENT DGM FOR SECOND ORDER ELLIPTIC BOUNDARY VALUE PROBLEMS

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Abstract. Since the analysis of nonlinear elliptic boundary value problem is through linearization, we, therefore, first discuss superconvergence results for nonselfadjoint linear elliptic problems using discontinuous Galerkin methods. The essential ingredient of the analysis is to construct an auxiliary projection which is based on the analysis of Cockburn et. al. [Math. Comp. 78(2009), pp. 1-24] for a selfadjoint linear elliptic equation. Then it is extended to derive superconvergence results for quasilinear elliptic problems. When piecewise polynomials of degree $k1$ are used to approximate both the potential as well as the flux, it is shown that the error estimate for the discrete flux in L^2-norm is of order k + 1. On solving a discrete linear elliptic problem at each element, a suitable postprocessing of the discrete potential is developed and then, it is proved that the resulting post-processed potential converges with order of convergence $k + 2$ in L^2-norm. These results confirm superconvergent results for linear elliptic problems.

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